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THESIS

ON SOME MARKOVIAN SALVO COMBAT MODELS

by

Say Beng Neo

December 2008

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In this thesis, we present Markov-based probability models for two important problems related to current combat situations: fire allocating of salvos against multiple targets, and Improvised Explosive Devices (IED) attacks on convoys transporting supply and troops. For the fire allocation problem, we suggest a certain shooting tactics, called *Persistent Shooting*, and explore the effect of various engagement parameters using a discrete time Markov chain. We consider the scenario where a single shooter engages a set of targets by a series of salvos. The shooter has a limited number of munitions to deliver and the question is how to allocate the fire in the presence of limited BDA capabilities. For the IED problem, we explore the effect of various tactical parameters on the IED threat and on the resulting attrition of the friendly force using a continuous time Markov chain.

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ON SOME MARKOVIAN SALVO COMBAT MODELS

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Submitted in partial fulfillment of the
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ABSTRACT

In this thesis, we present Markov-based probability models for two important problems related to current combat situations: fire allocating of salvos against multiple targets, and Improvised Explosive Devices (IED) attacks on convoys transporting supply and troops. For the fire allocation problem, we suggest a certain shooting tactics, called *Persistent Shooting*, and explore the effect of various engagement parameters using a discrete time Markov chain. We consider the scenario where a single shooter engages a set of targets by a series of salvos. The shooter has a limited number of munitions to deliver and the question is how to allocate the fire in the presence of limited BDA capabilities. For the IED problem, we explore the effect of various tactical parameters on the IED threat and on the resulting attrition of the friendly force using a continuous time Markov chain.

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TABLE OF CONTENTS

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LIST OF FIGURES

LIST OF TABLES

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LIST OF ABBREVIATIONS AND ACRONYMS

AGV	Autonomous Ground Vehicle
BDA	Battle Damage Assessment
CTMC	Continuous-Time Markov Chain
CREW	Counter- Radio Controlled Improvised Explosive Device Electronic Warfare
C4ISR	Command, Control, Communication, Computers, Intelligence, Surveillance, and Reconnaissance
EK	Evidently Killed
IED	Improvised Explosive Devices
IG	Infinitesimal Generator
K	Killed
L	Live
PS	Persistent Shooter
RS	Road Segment
SA	Situation Awareness
SLS	Shoot Look Shoot
UAV	Unmanned Aerial Vehicle
VT	Valuable Target
WT	Worthless Target

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LIST OF SYMBOLS

$E[k]$	Expected number of killed targets
k	Number of valuable targets in the cluster/Number of trucks in the convoy
p	Probability of targets is hit in each salvo/Probability of detecting and neutralizing IEDs
q	Probability of battle damage assessment/Conditional probability that a successful actuation of an IED removes a truck from the convoy
x	Number of salvo fired
n	Number of rounds fired in each salvo
M	Maximum "IED capacity" of the road segment
α	Probability of IED hitting a convoy
λ	IEDs placement rate
μ_1	Convoys dispatch rate
μ_2	IEDs detecting and neutralizing rate
σ	Reorder level at the destination point
η	Expected daily rate of attrition
θ	Demand rate for supply at the destination point

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EXECUTIVE SUMMARY

As technology progresses, battlefields are getting more complicated and the intensity of conflicts increases. There is an emerging trend to develop smart weapons that use passive and active sensors to accurately kill their targets without much collateral damage. However, most of these smart weapons deploy only a single payload to take out the intended target.

Contrary to the development of smart weapons by regular armed forces, insurgents and terrorists adopt tactics of asymmetric warfare, using relatively low-technology means that can be easily deployed by small groups diffused in the general population. Improvised Explosive Devices (IED) and suicide attacks are such tactics that are commonly used in Iraq and Afghanistan. These attacks are hard to predict, and a huge amount of intelligence is required to counter them effectively.

We develop two models aimed at obtaining insights about these two important issues related to current combat situations: fire allocating of salvos against multiple targets and IEDs attacks on convoys transporting supply and troops.

For the fire allocation problem, we consider a scenario where a shooter delivers a series of salvos onto a target area with the objective to kill a cluster of targets. The shooter has a limited number of munitions and has some prior intelligence about the type of targets in the area of operation. Before delivering a salvo, the shooter employs a sensor for detecting and collecting intelligence for better targeting. Based on the information obtained by the sensor, the shooter determines the fire allocation rule for the next salvo. This sequence of sensing and salvos repeats itself until the mission is over.

We develop a discrete time Markov chain model for the salvo shooting process to obtain some tactical insights regarding the effect of various engagement parameters. We show that for our shooting tactics (*persistent shooting*) and a certain set of realistic parameter values, delivering higher number of munitions in a salvo with smaller number of salvos is more effective than the reverse.

In the second problem, we consider a road segment where insurgents are placing IEDs to prevent friendly forces from transporting supply convoys through it. The friendly forces have IED-clearance units that patrol the road segment, searching for IEDs and attempting to neutralize them. There are three processes that determine the situation: (1) insurgents placing IEDs on the road segment, (2) friendly forces dispatching supply convoys that must cross the road segment and thus are vulnerable to IED attacks, and (3) IED-clearance units patrolling the road segment, detecting and neutralizing IEDs. We focus on the effects of different parameters on the attrition rate of a convoy passing through the road segment under two different scenarios:

- Basic operational scenario: Convoys and clearance teams traversing a road segment infested with IEDs
- Advanced logistical scenario: Basic operational scenario with inventory control that affects the traffic on the road segment.

We explore a range of operational settings by both the friendly forces and the insurgents in terms of convoy dispatch rates, IEDs placement rates, the probability of detecting and neutralizing the IED, the probability of hitting the convoys, and the maximum capacity of the number of IEDs present in the road segment. We wish to understand which parameters are important and where to invest to reduce the friendly force attrition rates.

We develop continuous-time Markov models to capture the key aspects that describe the situation and implement them computationally to obtain some tactical insights regarding the effect of various operational parameters on the outcome of convoy transportation missions in the presence of IEDs.

We show limiting the rate of convoys traveling through the road segment and at the same time increasing the rate of dispatching clearance units and improving the detecting and neutralizing capabilities will greatly reduce the attrition rate of convoys.

We also show that by increasing the number of trucks in each convoy, and thus reducing the frequency that convoys need to be sent out, and by fixing the order point in the destination level to a higher level by keeping larger inventory, will also reduce the attrition rate of the convoys.

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I. INTRODUCTION

A. BACKGROUND

As technology progresses in modern warfare, battlefields are becoming more complicated and the intensity of conflicts increases. Smart weapons utilize passive and active sensors to accurately kill their targets without much collateral damage. Such weapons are the subject of technological and operational research in many countries (Y. G. Liu & J. P. Hu, 1998) and (R. P. Hallion, 1997). Employing such weapons may involve using Unmanned Aerial Vehicles (UAV) to first search and detect the targets (M. Kress, & J. O. Royset, August 2007) and thereafter deploying smart weapons to kill them. Most smart weapons deploy a single payload to take out the intended target (Federation of Scientist, 2008).

Contrary to the development of smart weapons by regular armed forces, insurgents and terrorists adopt tactics of asymmetric warfare, using relatively low-technology means that can be easily deployed by small groups that are diffused in the general population. Improvised Explosive Devices (IED) and suicide attacks are such tactics that are commonly used in Iraq and Afghanistan. These attacks are hard to predict, and a huge amount of intelligence is required to counter them effectively.

In this thesis, we develop models aimed at obtaining insights about two important issues related to current combat situations: (1) fire allocating of salvos against multiple targets (e.g., insurgents' strongholds), (2) IEDs attacks on convoys transporting supplies and troops.

1. Salvos Allocation - Persistent Shooting Tactics

Precision weapons can engage targets with great accuracy and lethality, but to use these capabilities effectively weapons must rely on accurate and timely information generated by Command, Control, Communications, Computers, Intelligence, Surveillance, and Reconnaissance (C4ISR) systems. These systems not only allow a commander to have timely and accurate information and situational awareness (SA) of the battlefield, but they also provide a real-time live feed that gives the shooter the ability

to assess the damage inflicted on targets by previous shots and thus better utilize his weapon. This process of evaluating the condition of targets following an engagement is called Battle Damage Assessment (BDA).

However, with superior intelligence, SA and BDA are useless without the ability to strike targets with accuracy and lethality during the short window of opportunity normally present in the modern battlefield. Many of the targets in the modern battlefield are highly mobile and therefore they are time-sensitive or time-critical. In other words, once a target is acquired, the shooter has typically a small time window to engage it. Failure to kill the target during this time window will provide it an opportunity to hide or even counter fire at the shooter's location if the opponent also has good C4ISR capabilities. Therefore, weapon systems that can produce large and accurate salvos of fire are important in the modern battlefield; they may provide the edge over the opponent.

To look into this area, we develop a model for the *persistent* shooting tactics applied to a series of salvos, which generalizes the single shot persistent shooting tactics presented in (Y. Aviv, & M. Kress, 1997). We obtain some tactical insights regarding the effect of various engagement parameters. The *Persistent Shooter* uses shoot-look-shoot tactics and he might have imperfect BDA capabilities. The model is a discrete-time Markov chain.

2. Dispatching Convoys and Clearance Teams in the Presence of Improvised Explosive Device

The use and effect of Improvised Explosive Devices (IEDs) by insurgents against coalition forces in Iraq and Afghanistan has been of major concern (G. Zorpette, 2008; B. Hooffman, 2004; & D. W. Barno, 2007). These devices are normally homemade from readily available materials, ranging from explosive material to toxic chemicals. They are made by small groups who are trained in making and deploying IEDs.

IEDs pose the greatest threat to coalition forces deployed in Iraq and Afghanistan, killing 1811 coalition forces in Iraq and 261 coalition forces in Afghanistan as of November 2008 (Icasualties.org, 2008). Figure 1 shows the trend of coalition forces causalities since July 2004. The IED attacks normally happen along modern highways,

which are used for speedy movement of forces. The traffic pattern is observed by the insurgents and can be easily predicted. Outside of Iraq and Afghanistan, there are 200 to 350 IED attacks every month around the world (Hazard Management Solutions, 2008).

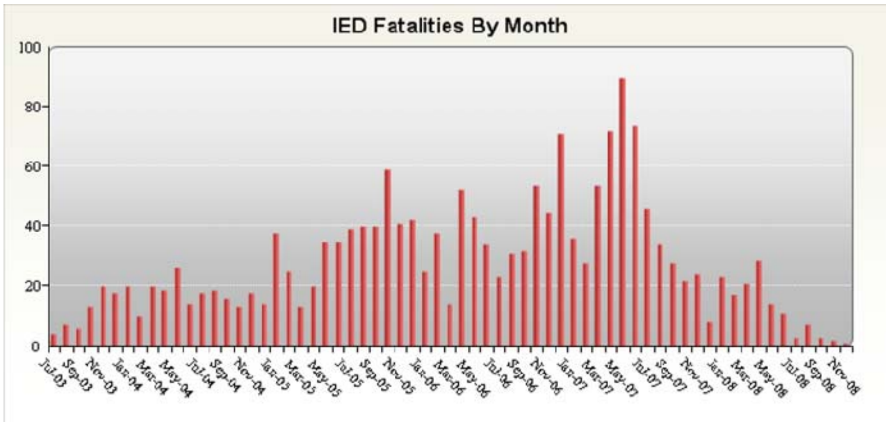

Figure 1. IED Fatalities by Month in Iraq (From: Icasualties.org, 2008)

The construction of IEDs is relatively simple and uses readily available low-technology devices such as mobile phones and car alarm systems. Using such means enables the shooter to be at a standoff position to fire the IED. However, with such low technology methods, an operator is normally required to observe the target. Figure 2 shows a typical IED setup along a road segment. An obstruction is normally used to slow down traffic so as to allow the target to stay in the kill zone for a longer period to increase the probability of kill.

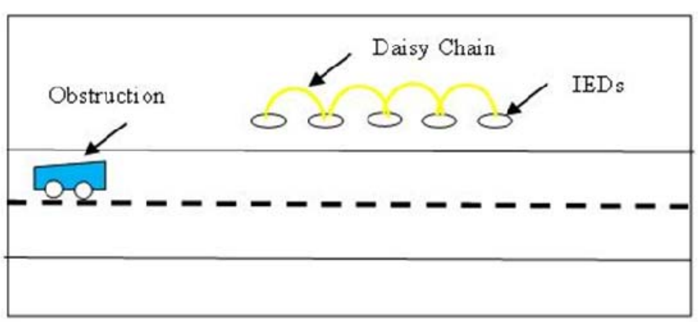

Figure 2. Typical setup of IED by insurgents.

Although the friendly forces are faced with these IED threats in day-to-day operations, there is still a need to send convoys to supply food and other necessities, as well as to move personnel for patrol and shift rotation. We develop models that capture key aspects of convoy transportation on roads infested with IEDs by using a continuous-time Markov chain (CTMC) that describes the situation and implement them to obtain some tactical insights regarding the effect of various operational parameters on the outcome of convoy transportation missions in the presence of IEDs.

B. PROBLEM STATEMENT

1. Shoot Look Shoot Tactics

We consider a Shoot-Look-Shoot (SLS) tactic where the engagement comprises a series of salvos. A deterministic analysis of some general SLS tactics for a single shooter can be found in L. B. Anderson, (1989) and Y. Aviv and M. Kress (1997). This thesis aims to study certain shooting tactics in these settings and to study their effectiveness. Specifically, we will focus on shoot-look-shoot tactics where each "shot" is a series of salvos of several rounds of fire aimed at a certain target. Following each salvo, a sensor is deployed to observe the target, perform BDA, and update the situational awareness of the shooter regarding the targets. Based on the updated picture of the target area, the shooter decides which target to engage next.

A target in the battlefield can be in three states: Live (L), Killed (K) or Evidently Killed (EK). A killed target may appear to be alive to the shooter if there are no signs to indicate otherwise. We assume that a previously killed target that was not evidently killed at the time of kill will not appear to the shooter as evidently killed later on unless it is engaged and "killed" again.

We consider the scenario where a shooter delivers a series of salvos onto a target area with the objective to kill Valuable Targets (VTs). The shooter has a limited number of munitions and has prior intelligence about the type of targets in the area of operation. Before delivering a salvo, the shooter employs a sensor for detecting and collecting intelligence for better targeting. Based on the information obtained by the sensor, the shooter determines the fire allocation rule for the next salvo. This sequence of sensing

("look") and salvos ("shoot") repeats itself until the mission is over. During this sequence, we assume that the target does not fire back at the shooter and therefore the shooter is not vulnerable.

2. Improvised Explosive Device

We consider a road segment (RS) in which the enemy is placing IEDs to prevent friendly forces from transporting supply convoys. The friendly forces have IED-clearance units that patrol the road segment, searching for IEDs and attempting to neutralize them. There are three processes that determine the situation: (1) The enemy placing IEDs on the road segment, (2) The friendly forces dispatching supply convoys that must cross the road segment and thus are vulnerable to IED attacks, and (3) IED-clearance units patrolling the road segment, detecting and neutralizing IEDs.

We focus on the effects of different parameters and how each of them will affect the attrition rate of the convoy passing through the road segment under two different scenarios.

- Basic operational scenario: Convoys and clearance teams traversing a road segment infested with IEDs
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We explore a range of operational setting by both friendly forces and the insurgents in terms of convoy dispatch rates, IEDs placement rates, the probability of detecting and neutralizing the IED, the probability of hitting the convoys, and the maximum capacity of the number of IEDs present in the road segment. We wish to understand which parameters are important and where to invest to reduce friendly force attrition rates.

C. LITERATURE REVIEW

1. BDA with Imperfect Sensors

The sensors that are used in warfare are imperfect and may cue targets with errors. The errors arise when a sensor has imperfect sensitivity, which leads to false negative results, and imperfect specificity, which leads to false positive detections. These

errors cause false targeting where real (valuable) targets are overlooked while false (worthless) targets, such as dummies or previously killed valuable targets, are acquired and engaged.

Because sensors are imperfect, it is typically difficult to get complete and accurate information about the presence, identity, and status of the targets in the area of operations. The probability of correctly detecting and assessing the status of a target may depend on the environment in the area of operations, the type of target, the type of weapon, and the type of sensor (Y. Aviv, & M. Kress, 1997). Furthermore, targets are dynamic and can move in and out of the area of operations, making the situation even more complex.

The importance of deploying good sensors to provide accurate damage assessment is evident in the recent Iraq and Afghanistan wars (Cordesman, 2004). There is an increasing trend in using UAVs, which provide an aerial view for detecting both threats and targets, to improve the friendly forces' BDA capabilities, largely in Iraq (Military.com, 2008). Although there is a great leap in the accuracy and precision of modern weapons, munitions fired at targets may still miss or cause only partial damage. This imperfection causes many problems in an urban setting as it may cause collateral damage to civilians and buildings. Good BDA capabilities will allow commanders to evaluate the situation correctly and therefore either fire more shots to kill the targets or save unnecessary shots that may otherwise expose the shooter for a longer time than necessary, making it vulnerable to enemy fire, and waste costly munitions that may result in logistical shortage.

2. Fire Engagement Sequence

The fire engagement of targets is usually broken down into three stages: *detection*, *acquisition*, and *fire*. In the detection stage, the weapon's sensor searches the area of interest for potential targets using a certain search pattern (M. Kress, & R. Szechtman, 2008). In an ideal situation, the sensor will be able to search and detect all potential targets—real (valuable) and false (worthless). However, in real life, some targets may be overlooked. In the acquisition stage, the shooter acquires targets that

appear to be valuable. A VT has a significant impact on the objective of the military operation, while a worthless target (WT) is a target with no operational value, a false target, or a VT that is already killed. In the acquisition stage, the sensor and the command and control system attempt to identify among the detected targets those which are valuable. The search optimization with imperfect specificity is discussed in Danskin (1962), A. Baggesen (2005) and M. Kress, K. Y. Lin and R. Szechtman (2007). Finally, in the fire stage, the shooter shoots at the acquired targets with the objective to kill them.

This three-stage process is repeated until one of the three following events occur: (1) a sufficient number of VTs are evidently killed, that is, the shooter is confident that the mission requirement for a number of killed VTs is satisfied, (2) the weapon runs out of ammunition, or (3) the mission is aborted because of operational reasons, such as threat to the shooter or reassignment of the shooter to another higher-priority mission. Finding the correct balance between sensors and shooters is a challenge and tradeoffs are required to optimize the use of assets (K. A. Yost & A. R. Washburn, 2000). The "Shoot Look Shoot" process is presented in the literature (L. B. Anderson, 1989; Y. Aviv & M. Kress, 1997; D. P. Gaver & P. A. Jacobs, 1997; K. A. Yost & A. R. Washburn, 2000; K. Glazebrook, & A. R. Washburn, 2004).

3. Insurgents and IEDs

Unmanned systems like Autonomous Ground Vehicle (AGV) has been developed by friendly forces (Defense Update, 2008) to provide surveillance to identify the human presence placing the IEDs and then report them to higher headquarters to take action (B. D. Miller, 2006). These systems patrol the area of operation autonomously to gather the intelligence. Guardium AGV shown in Figure 3 is one of the systems developed to do such tasks. Anther system that is currently in used in by the collation forces, shown in Figure 4, is the iRobot Packbot (Defense Update, 2008). It is normally used in defusing IEDs when they are detected.

Figure 3. Guardium UGV (From: G-NIUS, 2008)

Figure 4. Packbot (From: iRobot, 2008)

D. CONTRIBUTIONS OF THE THESIS

1. Persistent Shooter Model

With the emersion of new technologies in both the sensors and shooters arena, munitions are getting more lethal, accurate, and have a longer range. However, these weapons are very expensive to deploy and the shooter will want to ensure that every munition fired is killing a valuable target and not a target that is worthless or has already been previously killed.

After reviewing the literature of shoot-look-shoot shooting tactics, we notice that most of it involves a single-shot engagement process. The result obtained in this thesis for the salvo engagement process will allow commanders to increase their operational efficiency and utilization of the system through better decision-making when deploying their weaponry.

2. Improvised Explosive Device

A huge investment has been put into finding means and techniques to reduce the risk of IEDs attack to friendly forces, and some progress has been made. However, the constant threat of IEDs is still evident in Iraq and Afghanistan. The insights captured in this thesis will allow field commanders to make better decisions regarding investments in counter-IED technologies and in planning transportation operations

E. STRUCTURE AND METHODOLOGY

In Chapter II, we develop a discrete-time Markov chain model for the persistent salvo shooting process and obtain some tactical insights regarding the effect of various engagement parameters. The model is built on the single shooter *persistent* shooting tactics that is presented in Aviv and Kress (1997). Chapter III describes the situation and implementation of CTMC to obtain some tactical insights regarding the effect of various operational parameters on the outcome of convoy transportation missions in the presence of IEDs. Chapter IV builds on the base model discussed in Chapter III to capture key aspects of supply convoy transportation on roads infested with IEDs along with supply and demand inventory constraints by using a CTMC. Chapter V gives the conclusions, broad insights derived from the previous chapters, and a series of future work ideas.

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II. THE PERSISTENT SHOOTING TACTICS USING SALVOS

In this chapter, we develop a model for the *persistent* shooting tactics applied to a series of salvos instead of the single shooter that is presented in Y. Aviv, M. Kress (1997). The *Persistent Shooter* uses a shoot-look-shoot tactics and he might have imperfect BDA capabilities. We develop a discrete-time Markov chain model for a salvo shooting process following this tactic and obtain some tactical insights regarding the effect of various engagement parameters.

A. DESCRIPTION OF THE PROCESS

The shooting process begins when the shooter is assigned a set of VTs, which are targets that are threatening to the friendly force or have a high value to the enemy. The shooter engages the targets with a series of salvos. Following a salvo, the shooter conducts BDA to verify if the engaged targets are killed. Similar to Y. Aviv and M. Kress (1997), we assume that there are no false-positive errors, that is, a live target will never appear to the shooter as killed since a kill indication is usually obtained with certainty under extreme conditions (e.g., explosion or fire). It follows that if the shooter identifies a target as killed, it is *indeed* killed or *evidently killed* (EK). If a target is not identified as killed, which implies that it is either killed (but not EK) or still alive, then in the next salvo the shooter will engage it once again, hence the name *persistent shooter*. Fresh targets are engaged in salvo (n+1) only if some targets become EK in salvo n; EK targets are always replaced in the target set of a salvo by fresh targets. This process is repeated until either all targets are EK or the shooter runs out of ammunition. We assume that a killed, but not yet EK, target emits similar signs when hit as a live target and that the only way a killed target can become EK is if it is hit and "killed" again. To simplify the exposition we assume that the probability of a kill given a hit is 1. Thus, in order for a killed target to become EK it must be hit again.

B. PROBABILITY MODEL FOR THE SHOOTING PROCESS

In this section, we introduce a general Markov chain model for Persistent Shooting tactics when the engagement is conducted as a series of salvos. The shooter

selects the first set of targets to be engaged in a salvo size n out of a cluster of k targets at random and keeps engaging these targets as long as they are not EK. The state of the battle is described by the number of targets that are killed and evidently killed. These numbers are denoted by x and y, respectively.

The possible values for the number of killed targets, x, range between 0, when all targets are alive, and k, when all the targets are killed. Clearly, for a given x, the number of EK targets y is bounded above by x. The lower bound on y is determined by the value of x. If $x \le n$, then the lower bound on y is 0; however, if $x > n$ then, according to the engagement rule of the persistent shooter, the lower bound is $x - n$.

The total number of possible states is given by the sum of possible cases as discussed above. When $x \le n$, the possible states is given by $(n+1)\frac{n}{2}$, and when $x > n$ then the possible states number is $(k-n+1)(n+1)$.

Hence, the total number is:

$$(k-n+1)(n+1)+(n+1)\frac{n}{2} \tag{2.1}$$

$$=(n+1)\left(k-\frac{n}{2}+1\right)$$

Recall that a state in the Markov chain is defined by (x, y) where x is the number of killed target and y is the number of evidently killed targets. After each salvo, the state transition is defined by $(x, y) \rightarrow (x + i, y + j)$. To determine the transition probabilities, we need to consider two possible cases regarding y. In the first case, the number of targets yet to be engaged (not yet EK) is greater than the number of rounds in the salvo, that is, the sum of the number of killed targets that are not evidently killed and live targets is larger than n. Formally, $y \le k - n$. In the second case, the number of targets yet to be engaged is smaller than n. In the latter case, we assume that the size of the salvo is adjusted to the number of targets left and therefore the size of a salvo is smaller than n.

Case 1: $y \le k - n$. The $(x,y) \to (x + i, y + j)$ transition probability is

$$\binom{n+y-x}{i} p^i (1-p)^{n+y-x-i} \sum_{i=0}^{x-y} \binom{x-y}{l} p^l (1-p)^{x-y-l} \binom{l+i}{j} q^j (1-q)^{l+i-j} \tag{2.2}$$

where,

p – Probability of hit (=kill)

q – Probability that a hit is detected.

The transition probability in (2.2) is derived as follows. First, the probability of i new killed targets is obtained from a binomial distribution with parameters ($n + y - x$), which is the number of live targets in the salvo, and p, the hit (kill) probability. Second, given i new killed targets in that salvo, the probability of j new EK targets depends on i and on the number l of previously killed (but not EK) targets that have been hit again in that salvo. The latter number can be between 0 and (x-y) and also has a binomial distribution with parameters (x-y) and p.

Finally, given $i + l$ new hits, the number of EK indications among them is a binomial random variable with parameters ($i + l$) and q.

Case 2: $y > k - n$. The $(x,y) \to (x + i, y + j)$ transition probability is

$$\binom{k-x}{i} p^i (1-p)^{k-x-i} \sum_{i=0}^{x-y} \binom{x-y}{l} p^l (1-p)^{x-y-l} \binom{l+i}{j} q^j (1-q)^{l+i-j} \tag{2.3}$$

In this case, the number of new killed targets is bounded by k-x. Clearly, the state (k,k) is absorbing.

The case of $n = 1$ is studied in Aviv and Kress (1997). In that case the condition is

$$x - 1 \le y \le x \tag{2.4}$$

The probability distribution of the number of killed targets X is derived from the Negative Binomial distribution and is given by

$\Pr[X \ge x] = \sum_{i=x-1}^{n-1}$ Pr[The (x-1)th EK indication was obtained at the i-th round]*

Pr[At least one target is killed in the rest n-i rounds]

$$= \begin{cases} 1-(1-p)^n & x=1 \\ \sum_{i=x-1}^{n-1} \binom{i-1}{x-2} \alpha^{x-1}(1-\alpha)^{i-x+1}(1-(1-p)^{n-i}) & 1 < x \le k \\ 0 & x > k \end{cases}$$

where $\alpha = pq$.

C. ANALYSIS

The formulas for the transition probabilities given in Section B have been implemented in MATLAB to produce the analysis presented in this section. A base case with the following values is used to determine the effects of each parameter used in the equation above.

- Number of salvos fired, $x = 5$
- Number of round fired per salvo, $n = 2$
- Number of VT targets, $k = 5$
- Probability of kill in each salvo, $p = 0.5$
- Probability of Battle Damage Assessment, $q = 0.7$

The results and interpretation of the different setting of the analysis are as follows:

Case 1: Fixed: $p = 0.5$, $q = 0.7$, $n = 2$, $k = 5$
Varying: $x = [1, 30]$

Figure 5 shows that *E[k]* increases with x in a linear relationship in the initial values of 1 to 6. Thereafter, the graph has a nonlinear relationship that goes asymptotically towards 5. This is because the *E[k]* will reach the steady state of the total number of available VTs with a huge number of salvos being fired. However, in the battlefield, we normally do not need to eliminate all the VTs as killing up to a certain threshold will disable the enemy's effectiveness.

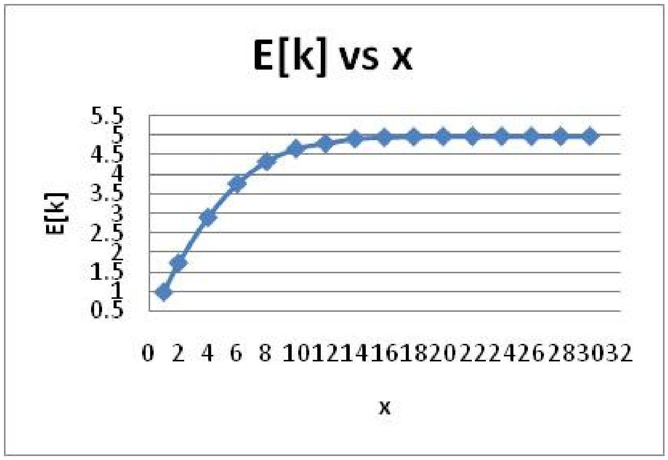

Figure 5. Plot of *E[k]* vs. *x*

Case 2; Fixed: $p = 0.5$, $q = 0.7$, $k = 5$, x = 5

Varying: $n = [2, 7]$

Figure 6 shows that *E[k]* increases with *n* in a nonlinear relationship. The value of *E[k]* goes to 4.4838 starting from $n = 5$.

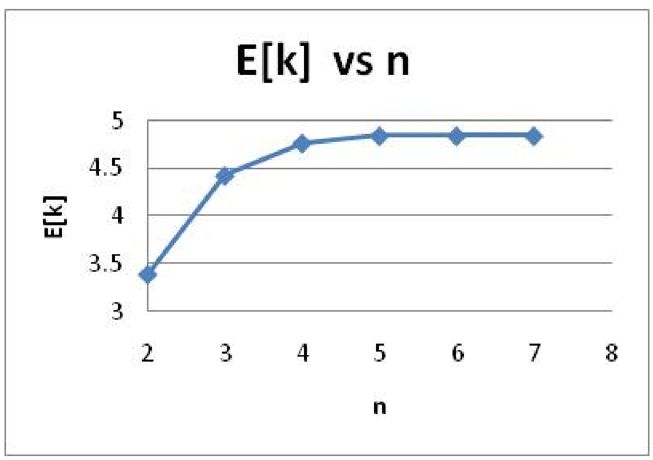

Figure 6. Plot of *E[k]* vs. *n*

Case 3: Fixed: $p = 0.5$, $q = 0.7$, $n = 2$, $x = 5$

Varying: $k = [1, 10]$

Figure 7 shows that *E[k]* increases with *k*, similar to *E[k]* vs. *n* where there is a nonlinear relationship between *E[k]* and *k* at the initial phase of the graph and it tapers towards a steady state value of 3.4688 when *k* is greater than 10.

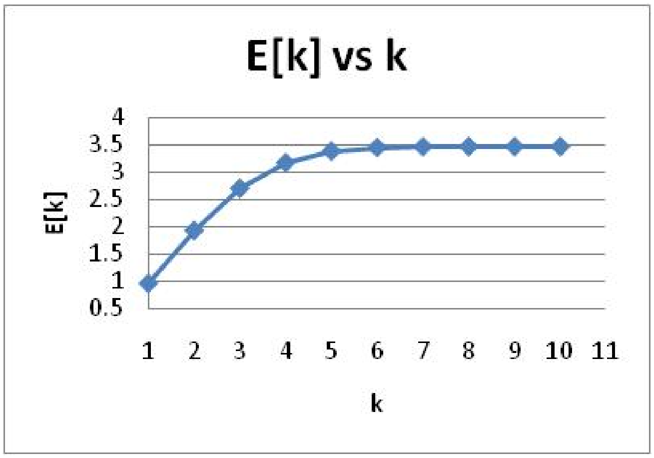

Figure 7. Plot of *E[k]* vs *k*

Case 4: Fixed: $q = 0.7$, $n = 2$, $k = 5$, $x = 5$

Varying: $p = [0, 1]$

Figure 8 shows that *E[k]* increases with *p*, with a maximum *E[k]* value of 4.8164 when p=1.

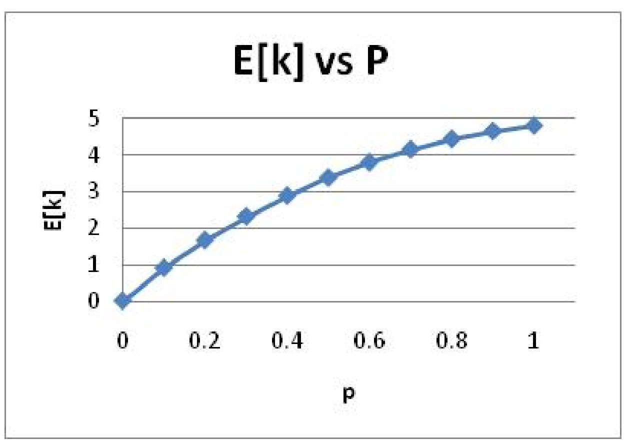

Figure 8. Plot of *E[k]* vs *p*

Case 5: Fixed: $p = 0.5$, $n = 2$, $k = 5$, $x = 5$

Varying: $q = [0, 1]$

Figure 9 shows that *E[k]* increases with *q*, there is a linear relationship between them at the initial value of *E[k]* which start off higher at 1.9375 when compared to the case when we are varying *p*. However, the value of *E[k]* with $q = 1$ has a lower value of 4.2832 compared to the case when *p* varies.

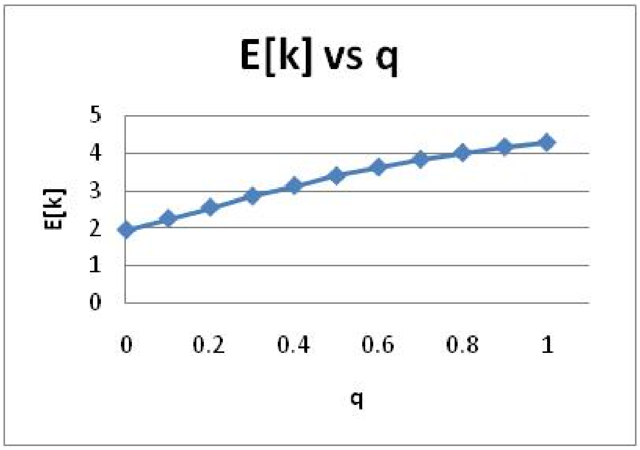

Figure 9. Plot of *E[k]* vs *q*

Case 6: Fixed: $p = 0.5$, $q = 0.7$, $k = 50$, total number of rounds available = 100.

Varying *n* and *k*.

For example, when the number of rounds used during each salvo, n = 2, the number of salvo, x = 100/2 = 50.

Figure 10 shows that the expected number of killed targets, *E[k]*, is increasing with the number of rounds used in each salvo, even though the rounds available in each case is fixed at 100. This increase in *E[k]* shows that it is favorable in situations when the available munitions in combat are limited to use a higher order number of salvos to kill the targets.

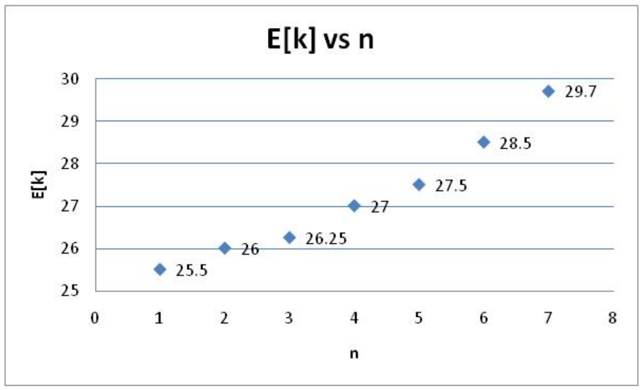

Figure 10. Plot of *E[k]* vs. *n*

III. TRANSPORTATION TACTICS IN THE PRESENCE OF IEDS

In this chapter, we develop models that capture key aspects of convoy transportation on roads infested with Improvised Explosive Devices (IEDs). We develop continuous-time Markov models (CTMC) that describe the situation and implement them computationally to obtain some tactical insights regarding the effect of various operational parameters on the outcome of convoy transportation missions in the presence of IEDs.

A. DESCRIPTION OF THE SITUATION

We consider a road segment (RS) in which the red side (insurgents/terrorists) is placing IEDs to prevent the blue side (friendly forces) from transporting supplies via convoys through that road. The blue side has IED-clearance units that patrol the RS, searching for IEDs and attempting to neutralize them. There are three processes that determine the situation: (1) Red placing IEDs on the RS, (2) blue forces dispatching supply convoys that must cross the RS and thus are vulnerable to IED attacks, and (3) blue IED-clearance units patrolling the RS, detecting, and neutralizing IEDs. These processes are governed by the following assumptions.

- Red places IEDs on the RS, one at the time, according to a Poisson process with parameter λ. The maximum IED-capacity of the road segment is *M*, that is, Red, who has perfect situational awareness, stops placing new IEDs if the number of IEDs on the road segment is *M*.
- Blue dispatches convoys, one at the time, according to a Poison process with rate μ_1. A convoy is subject to damage due to IEDs. The number of vehicles in a convoy, denoted by *k*, is larger than the maximum IED capacity *M*.
- Each IED is actuated independently by the convoy with probability α. A "successful" actuation of an IED removes one truck from the convoy with (conditional) probability, *q*. The actuations and removals are independent.
- Once a convoy is dispatched, it completes traversing the segment before any other event occurs.
- Blue dispatches clearance patrol units, one at the time, for detecting and neutralizing IEDs according to a Poisson process with rate μ_2

- The clearance team detects and neutralizes an IED with probability p and the detections/neutralizations are independent. The clearance team is not vulnerable to IED attacks.
- Once a clearance team is dispatched it completes traversing the segment before any other event occurs.
- The travel time of Blue units (convoys and clearance units) on the RS is short compared to the other temporal parameters. Thus, we assume that these units traverse the RS instantaneously. (See also Assumptions 4 and 7.) This assumption is relaxed later on.

B. CTMC MODEL

In this section, we introduce a continuous-time Markov model for the situation described in Section A. A *state* in this model is the number of IEDs planted along the road segment, m, where m = 0, 1, ..., M. Figure 11 shows the transition diagram for the case where M=3.

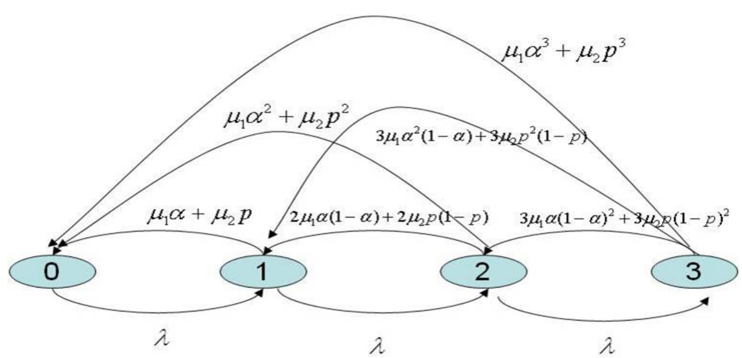

Figure 11. Birth-Death Process of the IEDs Model

Each node in Figure 11 represents a state and a left-to-right edge corresponds to a placement of an IED. The intensity that corresponds to each such edge is λ (see Assumption 1 in Section A). The right-to-left edges represent removals of IEDs by the friendly force either by detecting and neutralizing the IED by a clearance patrol unit or by actuating the IED by a convoy. From assumptions 2, 3, and 4 in Section A, the rate at which IEDs are removed from the segment are a combination of clearance and convoy-actuation rates. Specifically, if there are m IEDs present in the road segment, then the

removal rate depends on the deployment rates μ_1 and μ_2 of convoys and patrol units, respectively, and on the actuation probability (α) and the detection/neutralization probability (p) of these units, respectively.

We describe the process by its infinitesimal generator (IG) matrix. The off-diagonal entries of an IG matrix are the transition rates among the states of the process, while each entry on the diagonal is the negative sum of the other entries in that row. Thus, the sum of each row adds up to 0. We use the IG matrix to obtain the steady state probabilities as shown below in Equations 3.2 and 3.3. For details see (Minh, 2001). In our case, since a state is defined as the number of IEDs in the RS, the dimension of the IG matrix is M+1. The transition rates are:

(m) → (m+1) with transition rate λ

(m) → (i), i = 0, 1... m-1 with transition rate

$$\mu_1 \binom{m}{i} a^{m-i} \left(1-\alpha\right)^i + \mu_2 \binom{m}{i} p^{m-i} \left(1-p\right)^i \qquad (3.1)$$

The above transition rates generate the IG matrix of the placement, actuations and neutralization of the IEDS. Table 1 presents the IG matrix for the case M = 3 shown in Figure 11.

A =

m	***0***	***1***	***2***	***3***
0	$-\lambda$	λ	0	0
1	$(\mu_1\alpha + \mu_2 p)$	$-(\lambda+(\mu_1\alpha + \mu_2 p))$	λ	0
2	$(\mu_1\alpha^2 + \mu_2 p^2)$	$(2\mu_1\alpha(1-\alpha) + 2\mu_2 p(1-p))$	$-((\mu_1\alpha^2 + \mu_2 p^2)+ (2\mu_1\alpha(1-\alpha) + 2\mu_2 p(1-p))+ \lambda)$	λ
3	$(\mu_1\alpha^3 + \mu_2 p^3)$	$(3\mu_1\alpha^2(1-\alpha) + 3\mu_2 p^2(1-p))$	$(3\mu_1\alpha(1-\alpha)^2 + 3\mu_2 p(1-p)^2)$	$-((\mu_1\alpha^3 + \mu_2 p^3)+ (3\mu_1\alpha^2(1-\alpha) + 3\mu_2 p^2(1-p)) + (3\mu_1\alpha(1-\alpha)^2 + 3\mu_2 p(1-p)^2))$

Table 1. The IG Matrix for M=3

The steady state probabilities must satisfy:

$$\Sigma_m = 0.1....M^{\pi}{}_m = 1 \qquad\qquad \sum_{m=0}^{M} \pi_m = 1 \qquad (3.2)$$

Let $\hat{A}$ denote the IG matrix A where its last column is replaced by a column of 1's. The steady state probabilities are given by:

$$(\pi_0, \pi_1, ..., \pi_M) = (0, 0, ..., 1)\hat{A}^{-1} \tag{3.3}$$

We can now obtain the steady state rate of attrition η (expected number of trucks hit per unit time) by conditioning on the number of IEDs. Given m IEDs, with probability π_m the expected rate of casualties is $\mu_1 m\alpha q$. Thus,

$$\eta = \mu_1 \sum_{m=0}^{M} \pi_{\mathrm{m}} \; m\alpha q \tag{3.4}$$

Next, we wish to obtain the mean and variance of the number of trucks hit in a given time horizon $[0,t]$. Let $Z(t)$ be a random variable that counts the number of trucks hit by IEDs during the time interval. Note that

$$Z(t) = \begin{cases} \sum_{i=1}^{N(t)} X_i & \text{if } N(t) > 0 \\ 0 \quad \text{if} & N(t) = 0 \end{cases} \tag{3.5}$$

where $N(t)$ is the number of convoys dispatched during $[0,t]$, which is a Poisson random variable with mean and variance $\mu_1 t$, and X_i is the number of trucks hit in the *i*-the convoy.

At steady state X_i, $i = 1,...,N(t)$, are independently, identically distributed random variables with $E[X_i] = \alpha q \sum_{m=0}^{M} \pi_m m$ and

$$\begin{aligned} Var(X_i) &= E[Var(X_i|m)] + Var[E(X_i|m)] \\ &= \alpha q(1-\alpha q)\sum_{m=1}^{M} \pi_m m + (\alpha q)^2 \left(\sum_{m=1}^{M} \pi_m m^2 - \left(\sum_{m=1}^{M} \pi_m m \right)^2 \right). \end{aligned} \tag{3.6}$$

Next, we compute the mean and variance of $Z(t)$. Clearly $E[Z(t)] = E[N(t)]E[X_i] = \eta t.$ The variance of $Z(t)$ is

$$\begin{aligned} Var[Z(t)] = Var[\sum_{i=1}^{N(t)} X_i] &= E[N(t)]Var(X_i) + Var[N(t)](E(X_i))^2 \\ &= \mu_1 t(Var(X_i) + (E(X_i))^2) \end{aligned} \tag{3.7}$$

where $E(X_i)$ and $Var[X_i]$ are given above.

C. ANALYSIS

The model developed in Section B has been implemented in MATLAB to produce the analysis presented in this section. The following values are used as a base case for the analysis. The time resolution is in days.

- The maximum "IED capacity" of the segment, $M = 3$
- IEDs placement rate, $\lambda = 1$
- Convoys dispatch rate, $\mu_1 = 1$
- Probability of IED hitting a convoy, $\alpha = 0.5$
- IEDs detecting and neutralizing rate, $\mu_2 = 2$
- Probability of detecting and neutralizing IEDs, $p = 0.7$.
- Conditional probability that a successful actuation of an IED removes a truck from the convoy, $q = 0.5$.

First we note that the expected daily rate of attrition $\eta = 0.1278$ truck per day. The mean and variance of the number of trucks hit in a period of one month are:

$$E[Z(t)] = E[N(t)]E[X_i] = \eta t.$$

$$= 3.84$$

$$Var[Z(t)] = Var[\sum_{i=1}^{N(t)} X_i] = E[N(t)]Var(X_i) + Var[N(t)](E(X_i))^2$$

$$= \mu_1 t(Var(X_i) + (E(X_i))^2)$$

$$= 4.96$$

The values show that on the average there will be about 4 trucks being hit by IEDs every month with a standard deviation of about 2.2 trucks per month. Therefore, the P [Z (t) ≥ 5] can be estimated from the Central Limit Theorem as follows.

$$\begin{aligned} P\left[Z(t) \geq 5\right] &= P\ [Z \geq \frac{x-\mu}{\sigma}] \\ &= P\ [\ Z \geq \frac{5-3.84}{2.2}] \\ &= P\ [\ \ Z \geq 0.527] \\ &= 0.3 \end{aligned}$$

In the following, we perform sensitivity analysis with respect to the base case**.**

Case 1: Varying the IED capacity of the RS. M = [1, 10]

Figure 12 shows that the expected rate of attrition η increases with M in a non-linear fashion. The value of η when $M = 1$ is 0.0826, while the values of η for M greater than 5 level off at 0.1316. This behavior of η shows that the steady state attrition rate is saturated at $M = 5$. Thus increasing the capacity of the RS beyond 5 IEDs by Red will not affect the expected rate of Blue's attrition. This phenomenon is due to the relatively high IED clearance rate (both by convoys and clearance units) compared to the placement rate of IEDs by Red. Thus, although the capacity is high, it is never reached. Therefore, the steady state attrition rate is saturated and is given by $\frac{\lambda}{\mu_1 \alpha + \mu_2 p} \alpha q$, which is 0.1316.

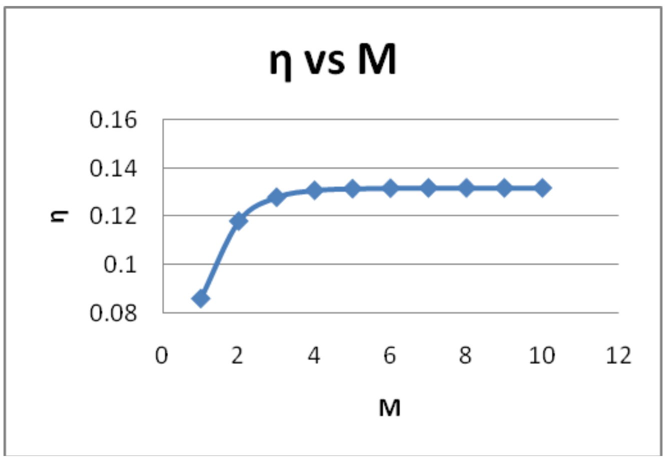

Figure 12. Plot of η vs. M

Case 2: Varying the rate at which Red is planting IEDs. λ = [0.5, 3]

Figure 13 shows that in this range of λ, η increases almost linearly with the rate at which Red plants IEDs. For example, if Red doubles the intensity of attacks from one IED a day to two IEDs a day, then the expected rate of attrition increases from 0.1278 a day to 0.2347 a day.

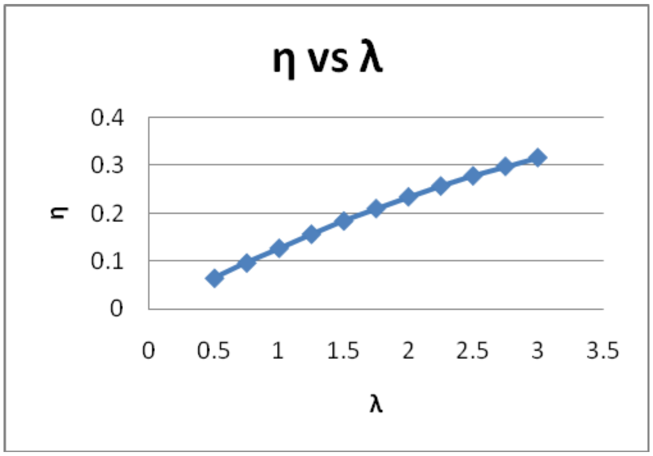

Figure 13. Plot of η vs. λ

Case 3: Varying the rate at which Red is planting IEDs. λ = [0, 36]

Figure 14 is an expansion of Case 2 with the range of λ increasing from 0 to 36. The plot shows that while initially η increases with λ almost linearly. This relationship eventually levels off and approaches asymptotically to 0.75. The asymptotic value is equal to $M\alpha q$. When Red can plant IEDs almost instantly, there will always be M IEDs on the RS and therefore the expected number of trucks hit by IEDs is the expected value of a Binomial distribution with parameters M and αq.

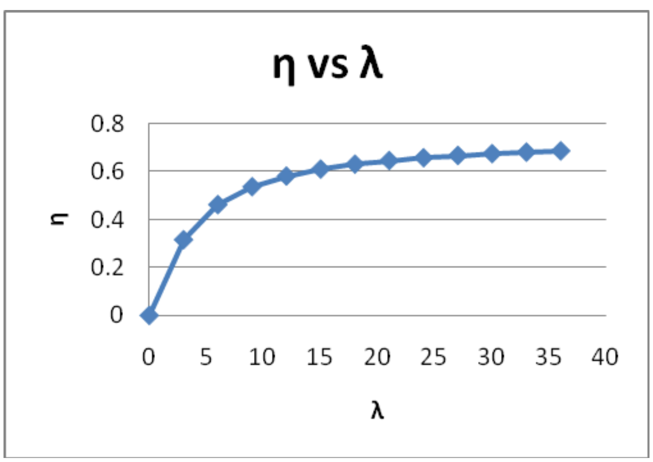

Figure 14. Plot of η vs. λ

Case 4: Varying the dispatch rate of convoys. μ_1 = [0.25, 3]

Figure 15 shows that in this range of μ_1, η increases almost linearly with the rate at which Blue dispatches its convoys. If Blue doubles the rate at which it sends its convoys from one truck a day to two trucks a day, then the expected rate of casualties increases from 0.1278 a day to 0.2049 a day. Therefore, it is crucial to only schedule a convoy when there is a need. The number of convoys passing the road segment should be minimized.

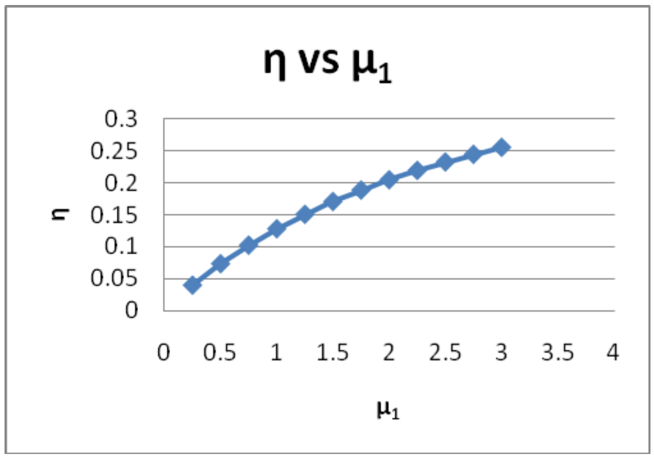

Figure 15. Plot of η vs. μ_1

Case 5: Varying the dispatch rate of convoys. μ_1 = [0, 33]

Figure 16 is an expansion of Case 4 with the range of μ_1 increasing from 0 to 33. The plot shows that while initially η increases with μ_1 almost linearly, this relationship eventually levels off and approaches asymptotically 0.5. If the convoys are dispatched very frequently, then each planted IED is immediately actuated and a fraction q of these actuations is effective, thus, $\eta \to \lambda q = 1 \times 0.5 = 0.5$.

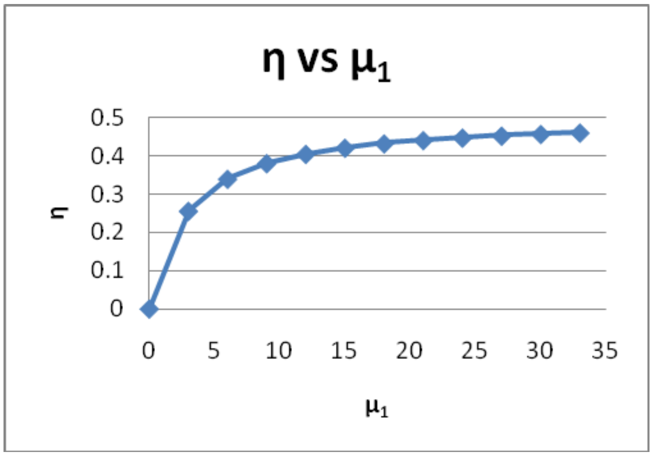

Figure 16. Plot of η vs. μ_1

Case 6: Varying the IEDs detecting and neutralizing rate, μ_2 = [0.5, 4]

Figure 17 shows that in this range of μ_2 η decreases exponentially with the rate at which Blue dispatches the clearance units. If Blue doubles the rate of dispatching these teams from one team a day to two teams a day then the expected rate of casualties decreases significantly from 0.1937 a day to 0.1278 a day.

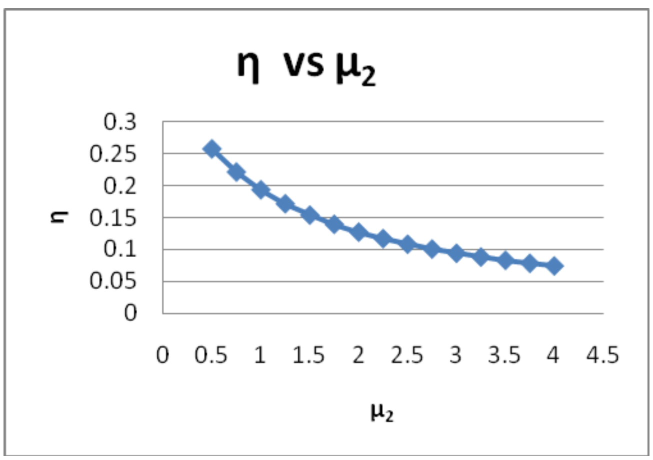

Figure 17. Plot of η vs. μ_2

Case 7: Varying the IEDs detecting and neutralizing rate, μ_2 = [0, 33]

The expansion of Case 6 for the range of μ_2 from 0 to 33 is shown in Figure 18. We can see that the value of η is approaching 0 as μ_2 goes to ∞. This phenomenon happens because the rate at which the clearance teams are clearing the IEDs is much faster than the rate the Red is deploying the IEDs in the RS. Thus, the convoys face no threat. The significant drop in the expected rate of casualties is when μ_2 is smaller than 5. Beyond this value, the effect is marginal.

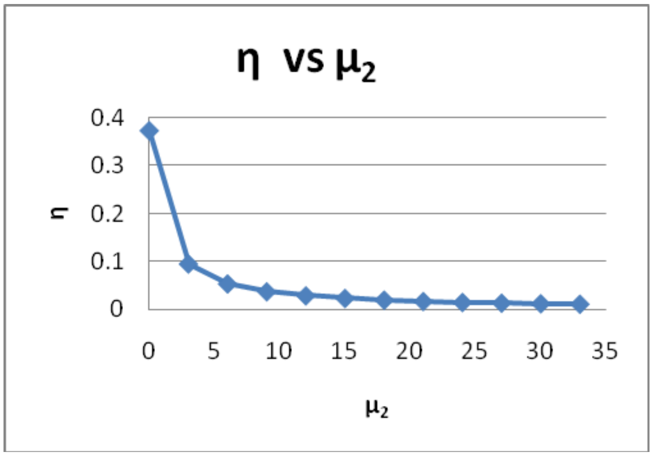

Figure 18. Plot of η vs. μ_2

Case 8: Varying the probability of IED hitting a convoy, α = [0, 1]

Figure 19 shows that η increases with α in almost a linear relationship. The expected rate of casualties increases by about 0.025 when the probability of the IED hitting a convoy increases by 0.1. We note that although this parameter does not contribute significantly to the expected rate of casualties, in practice the coalition forces invest quite a lot in jamming devices that are intended to reduce this parameter (G. Zorpette, September 2008).

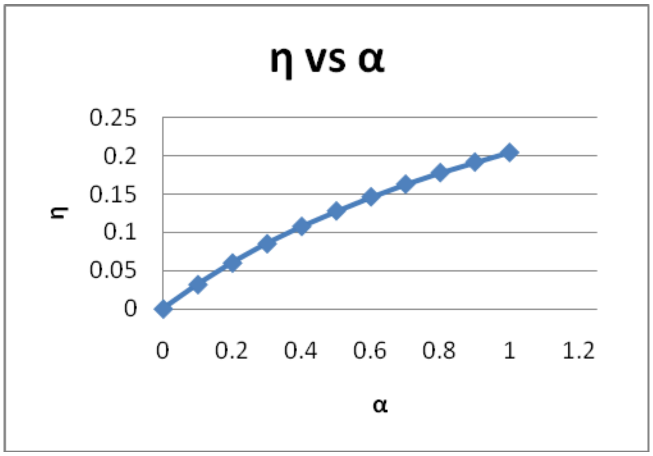

Figure 19. Plot of η vs. α

Case 9: Varying probability of detecting and neutralizing IEDs, $p = [0, 1]$

Similarly to the result shown in Case 7, if the probability of detecting and neutralizing the IED increases, the steady state rate of attrition will decrease significantly. However, the rate that expected rate of casualties decreases a slower rate compared to case 7. Figure 20 shows that if Blue doubles the probability of detecting and neutralizing the IEDs from 0.2 to 0.4, then the expected rate of casualties decreases from 0.2503 a day to 0.1823 a day.

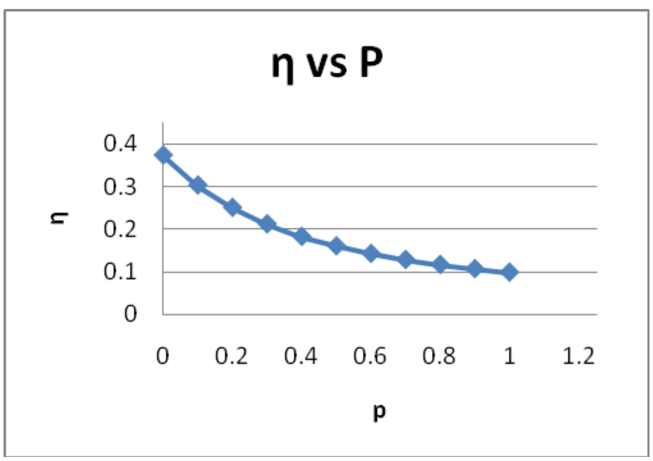

Figure 20. Plot of η vs p

Case 10: Varying: μ_1, μ_2 = [1, 15]

Figure 21 shows that the expected rate of casualties increases when the intensity at which Blue is sending out convoys is increasing and the intensity of sending out clearance units is small. For example, the highest expected rate of causalities is 0.4571 a day when the μ_1 = 15 and μ_2 = 1, compared to 0.0227 a day when μ_1 = 1 and μ_2 = 15. Therefore, it is crucial to only schedule convoys when there is a need and to increase the rate at which clearance teams are dispatched.

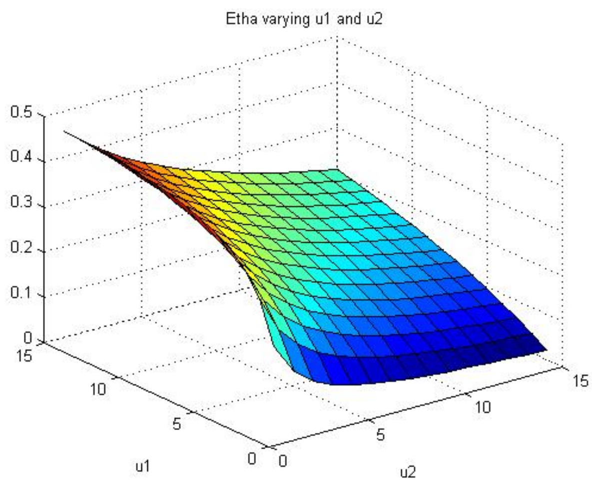

Figure 21. Plot of η vs. μ_1 and μ_2

Case 11: Varying: p, α = [0, 1]

Figure 22 shows that the expected rate of causalities has the highest value when the probability of an IED hitting a convoy , α =1 and the probability of detecting and neutralizing IEDs, $p = 0$. For example, the highest expected rate of causalities is 0.4375 a day when the probability of IED hitting a convoy is 1 and the probability of detecting and neutralizing is 0 compared to the rate 0 a day when α =0. Given the two options with respect to the base case, to reduce α by 20% or increase p by 20%, it will be more beneficial to reduce α as the rate of change in the slope is faster than p, especially at the lower values.

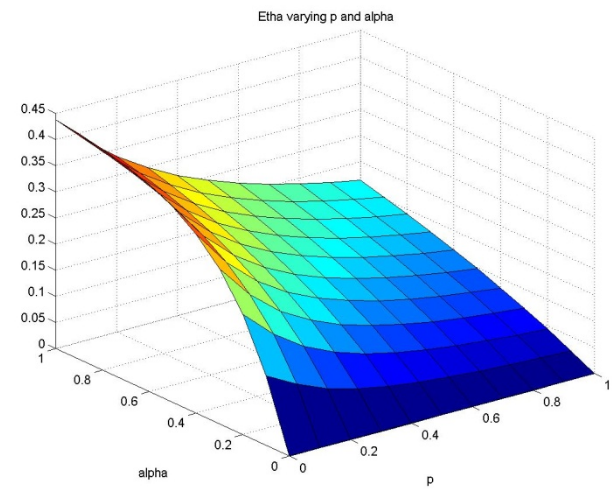

Figure 22. Plot of η vs. p and α

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IV. A SUPPLY MODEL IN THE PRESENCE OF IEDS

In this chapter, we build on the base model that was discussed in Chapter 3 to capture key aspects of supply convoy transportation on roads infested with Improvised Explosive Devices (IEDs). We develop a continuous-time Markov model (CTMC) that describes the situation and implement it to obtain some tactical insights regarding the effect of various tactical parameters that affect the outcome of supply convoy transportation missions.

A. DESCRIPTION OF THE SITUATION

We consider a RS in which Red places IEDs to prevent Blue from transporting supply convoys through that road. The processes and assumptions that are governing the placing, detecting, and neutralizing of IEDs and dispatch rate of convoys are the same as described in Part A of Chapter III. Blue's destination point has a demand that consumes the supply according to the Poisson process with the parameter θ. The supply level in the destination point is denoted by *s* and has an ordering point, denoted by σ. The demand is measured in truckloads. Once the supply level at the destination point is less than or equal to the order point, an order is placed and Blue will dispatch its convoy, comprising *k* trucks, after a setup time that has exponential distribution with parameter μ_1. No convoy is dispatched if the inventory in the destination point is higher that the ordering point, σ.

B. CTMC MODEL

In this section, we introduce a continuous-time Markov model for the situation described in Section A. The *state* of the model is described by the number of IEDs in the RS and the supply level at the destination point. These numbers are denoted by *m* and *s* respectively.

The possible values for the number of IEDs in the RS, *m,* range between 0, when there are no IEDs to *M,* when the number of IEDs in the RS is at its maximum. The possible values for the supply level at the destination point, *s*, ranges between 0, when the inventory is totally depleted, and σ + k*,* when the inventory is at its ordering threshold and *k* trucks arrive at the destination point.

The total number of possible states (*m, s*) is given by the product of the possible number of IEDs, *M*+1, and the $\sigma + k + 1$ possible inventory levels at the destination point: $(M+1)(\sigma + k + 1)$, which is also the dimension of the corresponding infinitesimal generator (IG) matrix. The transition rates are given in the following cases.

Case 1: (*m, s*) → (*m+1, s*) with transition rate λ

This happens when there is no consumption at the destination point and the enemy places one IED.

Case 2: (*m, s)* → (*m, s-1*) with transition rate θ

This happens when there is consumption of one supply unit at the destination point and Red did not place any IEDs on the RS.

Case 3: $s > \sigma$

(*m, s*) → (*m-i, s*)

i = 1, 2..., *m* with transition rate

$$\mu_2 \binom{m}{i} p^i (1-p)^{m-i} \tag{4.1}$$

When the supply level, *s*, is greater than the ordering point, σ, no order is placed and thus no convoy is dispatched. The only possible state change is when the clearance team manages to detect and neutralize IEDs in the RS.

Case 4: $s \le \sigma$ `

(*m, s*) → (*m-i , s*),

i = 1, 2,....., *m* with transition rate

$$\mu_2 \binom{m}{i} p^i (1-p)^{m-i} \tag{4.2}$$

This transition occurs when the next event is dispatching a clearance team, which detects and clears i IEDs. This transition is the same as in Case 3.

$(m, s) \rightarrow (m\text{-}i, s\text{+}k\text{-}j)$,

$i = 0, 1, 2....., m$

$j = 0, 1, 2......., i$ with transition rate

$$\mu_1 \binom{m}{m} \alpha^i (1-\alpha)^{m-i} \binom{i}{j} q^j (1-q)^{i-j} \tag{4.3}$$

When the supply level s is less than or equal to the order point σ, an order is triggered and a convoy of k trucks is sent to the destination point to meet the demand. This transition occurs if and only if the next event is a dispatching of a convoy. In that event i IEDs are actuated, with probability $\mu_1 \binom{m}{i} \alpha^i (1-\alpha)^{m-i}$ out of which j actuations are damaging, with probability equal to $\binom{i}{j} q^j (1-q)^{i-j}$. The remaining k-j trucks successfully deliver the k-j units of supply to the destination point.

The above transition rates generate the IG matrix of the placement, actuation and neutralization of the IEDS and the supply level as discussed above. Table 2 presents the IG matrix for the case $M = 1$, $\sigma = 1$ and $k = 2$.

$A =$

(m,s)	**00**	**01**	**02**	**03**	**10**	**11**	**12**	**13**
00	$-(\mu_1+\lambda)$	0	μ_1	0	λ	0	0	0
01	θ	$-(\theta+\mu_1+\lambda)$	0	μ_1	0	λ	0	0
02	0	θ	$-(\theta+\mu_1+\lambda)$	0	0	0	λ	0
03	0	0	θ	$-(\theta+\mu_1+\lambda)$	0	0	0	λ
10	$\mu_2 p$	$\mu_1(\alpha)(q)$	$\mu_1(\alpha)(1-q)$	0	$-(\mu_2 p + \mu_1(\alpha)(q) + \mu_1(\alpha)(1-q) + \mu_1(1-\alpha))$	0	$\mu_1(1-\alpha)$	0
11	0	$\mu_2 p$	$\mu_1(\alpha)(q)$	$\mu_1(\alpha)(1-q)$	θ	$-(\theta + \mu_2 p + \mu_1(\alpha)(q) + \mu_1(\alpha)(1-q) + \mu_1(1-\alpha))$	0	$\mu_1(1-\alpha)$
12	0	0	$\mu_2 p$	0	0	θ	$-(\mu_2 p + \theta)$	0
13	0	0	0	$\mu_2 p$	0	0	θ	$-(\mu_2 p + \theta)$

Table 2. The IG Matrix for M=1, σ=1 and k=2

The steady state probabilities must satisfy:

$$\Sigma_m = 0.1....M^{\pi} m = 1 \quad \sum_{m=0}^{M} \pi_m = 1 \tag{4.4}$$

Let $\hat{A}$ denote the IG matrix A where its last column is replaced by a column of 1's. The steady state probabilities are given by:

$$(\pi_0, \pi_1, ..., \pi_M) = (0, 0, ..., 1)\hat{A}^{-1} \tag{4.5}$$

C. ANALYSIS

The model developed in Section B has been implemented in MATLAB to produce the analysis presented in this section. The following values are used as base case to determine the effects of each parameter used in the model.

- The maximum "IED capacity" of the segment, $M = 3$
- IEDs placement rate, $\lambda = 1$
- Convoys dispatch rate, $\mu_1 = 1$.
- Probability of IED hitting a convoy, $\alpha = 0.5$.
- IEDs detecting and neutralizing rate, $\mu_2 = 2$.
- Probability of detecting and neutralizing IEDs, $p = 0.7$
- Conditional probability of successful actuation of an IED removing a truck from the convoy, $q = 0.5$
- Demand rate for supply at the destination point, $\theta = 1$
- Reorder level at the destination point, $\sigma = 3$
- Number of trucks in the convoy, $k = 5$

Figure 23 presents the long-run Probability Mass Function (PMF) of the number of active IEDs on the RS, varying from 0 to 3 (=M).

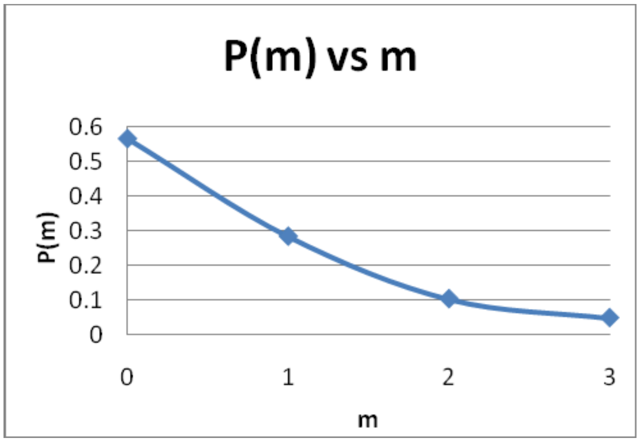

Figure 23. Plot of the PMF *P (m)* vs. *m*

The long-run PMF *P(s)* of the possible values for the supply level at the destination point is shown in Figure 24. The supply level *s* is varied from 0 to 8 (= σ + *k*).

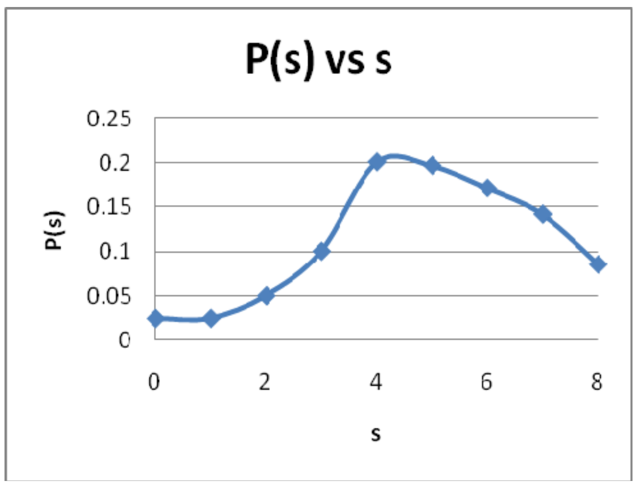

Figure 24. Plot of *P(s)* vs. *s*

From the plots above, we can see that the *P (m)* is decreasing as *m* increases, which means that for the case when *M* is 3, the long-run probability of having no IEDs in the RS is the highest. The highest value of *P(s)* is when *s* is 4. This is the point that is right after the reorder point of 3. *P(s)* starts to decrease as *s* increases from 5 to 8.

In the following, we perform sensitivity analysis with respect to the base case. The time resolution is in days**.**

Case 1: Varying the IED capacity of the RS, M= [1, 4]

Figure 25 shows that the long-run expected number of IEDs increases from 0.3994 when $M = 1$ to 0.6533 when $M = 4$. It has the same trend as the Case 1 in Chapter 3. The expected number of IEDs is increasing when the maximum "IED capacity" of the segment is increasing as well. However, Figure 26 shows that the long-run expected number of supply units in the destination point is decreasing as M increases. The long run expected number of supply units, $E(s)$, decreases from 4.956 units when $M = 1$ to 4.9297 units when $M = 4$. Evidently, this is not a significant decrease in the average supply level at the destination point and thus the IED capacity has little logistical effect.

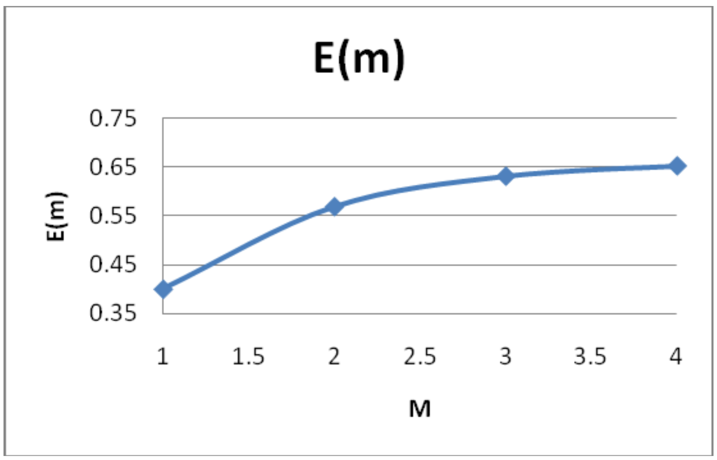

Figure 25. Plot of $E(m)$ vs. M

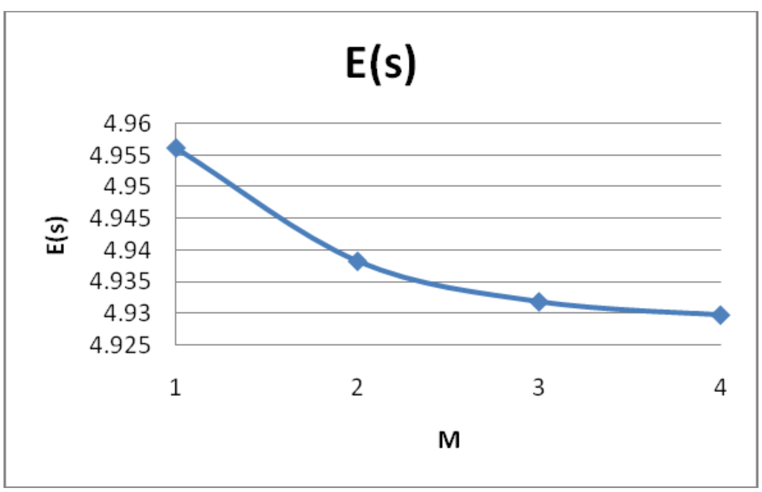

Figure 26. Plot of $E(s)$ vs. M

Case 2: Varying the number of trucks in a convoy, $k = [4, 7]$

Figure 27 shows that the expected number of IEDs slightly increases from 0.622 when $k = 4$ to 0.6439 when $k = 7$. At a first look this behavior seems counter intuitive as we would expect $E(m)$ to decrease when the number of trucks passing through the RS increases. However, this trend may be explained by the fact that longer convoys result in a smaller number of dispatches, and thus fewer convoys-IED encounters.

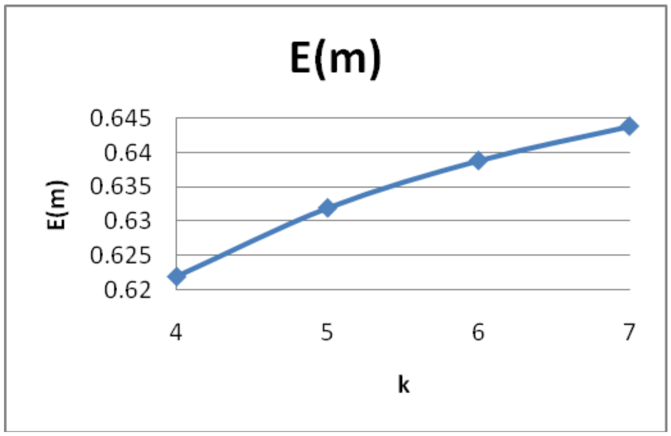

Figure 27. Plot of $E(m)$ vs. k

Figure 28 shows that the long-run expected number of supply units in the destination point is increasing as the number of trucks in the convoy increases. The expected number of supply units increases from 4.419 unit when $k = 4$ to 5.945 unit when $k = 7$.

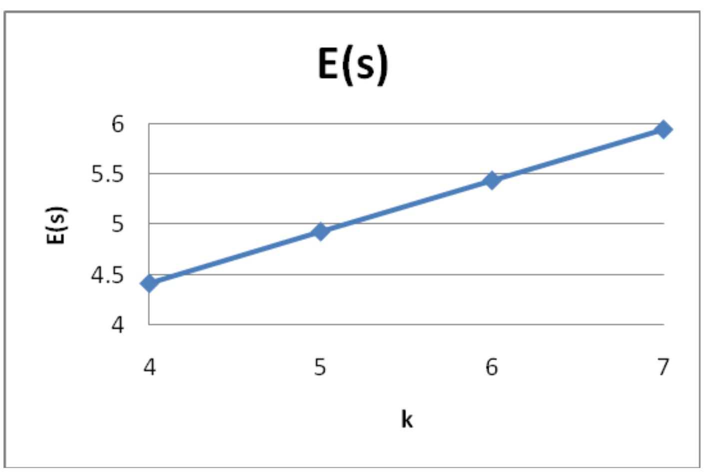

Figure 28. Plot of $E(s)$ vs. k

Case 3: Varying IEDs placement rate, λ = [0.25, 2]

Figure 29 shows that the long run expected number of IEDs increases when the IEDs placement rate λ increases. *E(m)* increases significantly from 0.1659 when λ = 0.25 to 1.1161 when λ = 2. Figure 30 shows that the expected number of supply units decreases as IEDs placement rate λ increases. The expected number of supply units decreases from 4.982 unit when λ = 0.25 to 4.88 unit when λ = 2. In both plots, the relations are almost linear as the number of IEDs in the RS has not reached the maximum capacity, as shown in Case 3 in Chapter III.

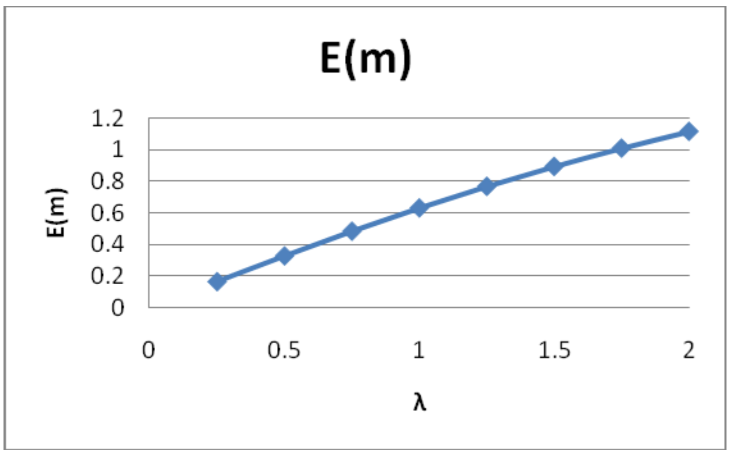

Figure 29. Plot of *E(m)* vs. λ

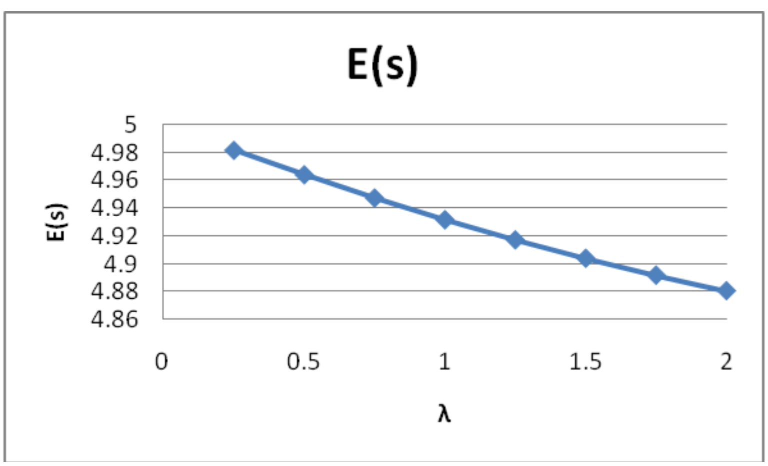

Figure 30. Plot of *E(s)* vs. λ

Case 4: Varying convoys dispatch rate, μ_1 = [0.5, 4]

Figure 31 shows that in this range of μ_1, the long run expected number of IEDs in the RS is initially decreasing with μ_1 almost linearly, and this relationship eventually levels off and approaches asymptotically to 0.63. If Blue doubles the rate at which it sending its convoys from one truck a day to two trucks a day, then the long-run expected number of IEDs decreases from 0.632 to 0.6308. This is not a significant increase as the demand is not high. However, when there is unlimited demand rate for supply at the destination point, *E(m)* in the RS will be decreasing as shown the plot below.

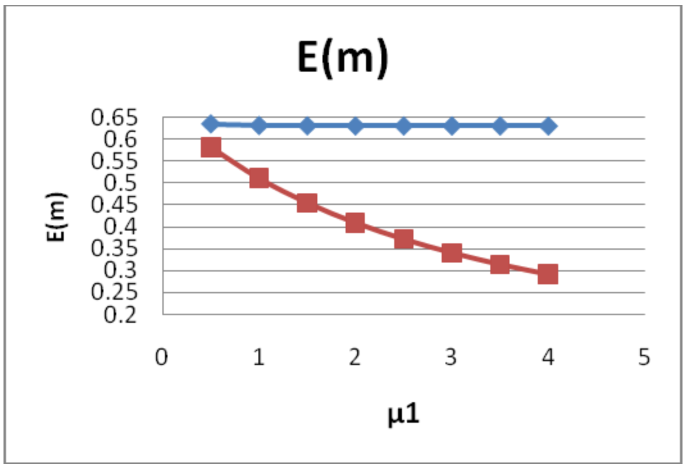

Figure 31. Plot of *E(m)* vs. μ_1

Figure 32 shows that in this range of μ_1, the expected number of supply units is initially increasing with μ_1 almost linearly; this relationship eventually levels off and approaches asymptotically to 8, which is $\sigma + k$. If Blue doubles the rate at which it is sending its convoys from one truck a day to two trucks a day, then the expected number of supply increase from 4.932 unit to 5.533 unit.

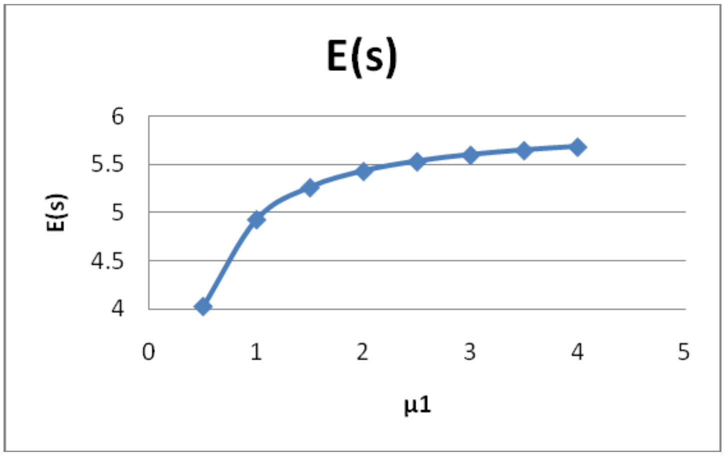

Figure 32. Plot of *E(s)* vs. μ_1

Case 5: Varying IEDs detecting and neutralizing rate, μ_2 = [0.5, 4]

Similar to Case 4, Figure 33 shows that in this range of μ_2, the long run expected number of IEDs in the RS is initially decreasing with μ_2 almost linearly, this relationship eventually levels off and approaches asymptotically to 0. If the detection and neutralization rate are much faster than the Red IEDs deployment rate, the *E(m)* will eventually become insignificant. With these results, the *E(s)* shown in Figure 34 will eventually go to 5 units at steady state.

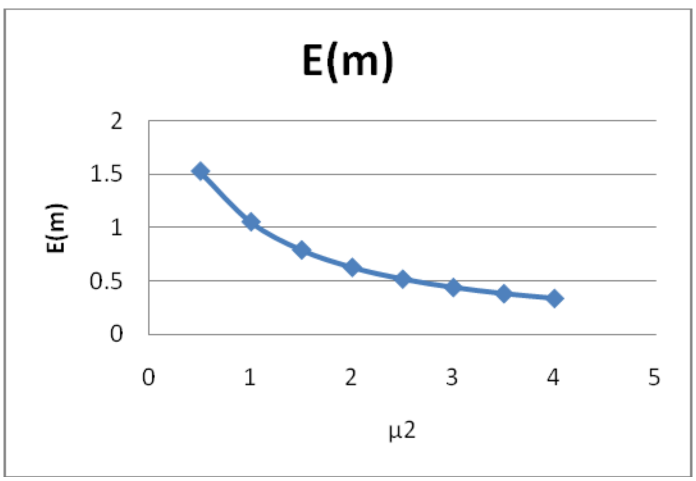

Figure 33. Plot of *E(m)* vs. μ_2

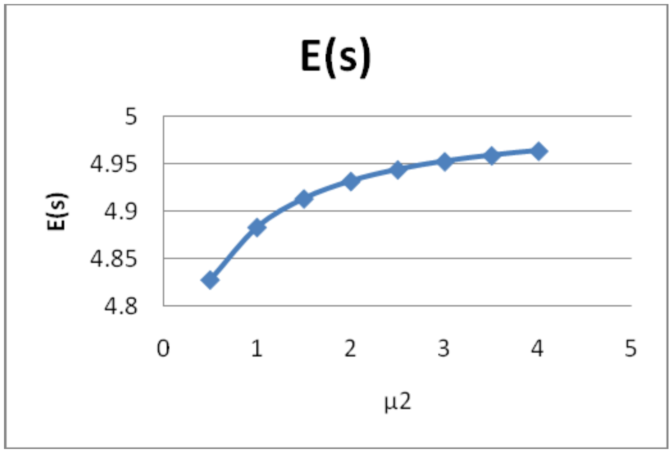

Figure 34. Plot *of E(s)* vs. μ_2

Case 6: Varying order point in the destination point: σ = [1, 5]

Figure 35 shows that $E(m)$ decreases with reorder level σ at the destination point, given that the maximum IED capacity in the RS is 3, the long run expected number of IEDs in the RS will start to level off when the order point is beyond the IED capacity. As shown in the plot, the *E(m)* decreases from 0.6348 when σ =1 to 0.6315 when σ = 5. However, the *E(s)* is increasing when the order point in the destination point, σ is increasing. The value of *E(s)* increases from 3.112 unit to 6.9157 a day when σ is increased from 1 to 5.

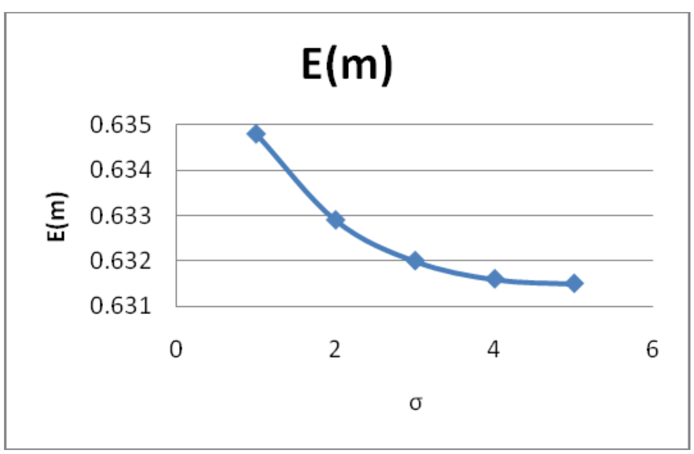

Figure 35. Plot of *E(m)* vs. σ

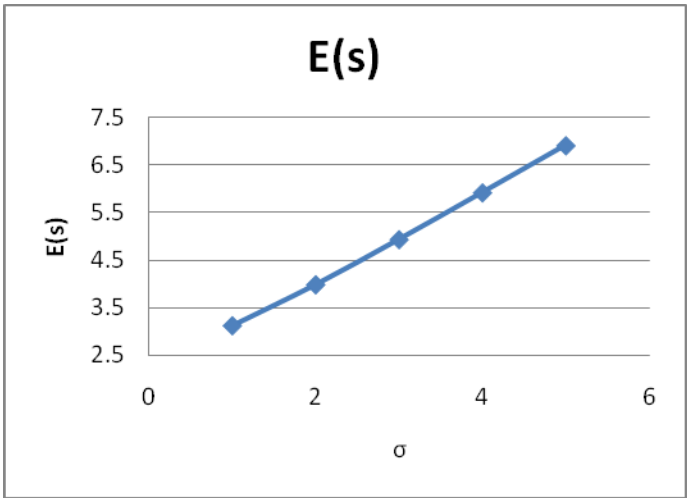

Figure 36. Plot of $E(s)$ vs. σ

Case 7: Varying friendly force's destination point inventory demand rate, θ = [0.5, 4]

Figure 37 shows that $E(m)$ decreases with Blue's destination point inventory demand, θ. As shown in the plot, the $E(m)$ decreases from 0.632 when θ =1 to 0.5613 when θ = 4. This trend is due to Blue sending more convoys to satisfy the destination point demand. However, even though there are more convoys being sent, Figure 38 shows that $E(s)$ decreases with Blue's destination point inventory demand, θ. Its value decreases from 5.4318 when θ =1 to 2.804 when θ = 4.

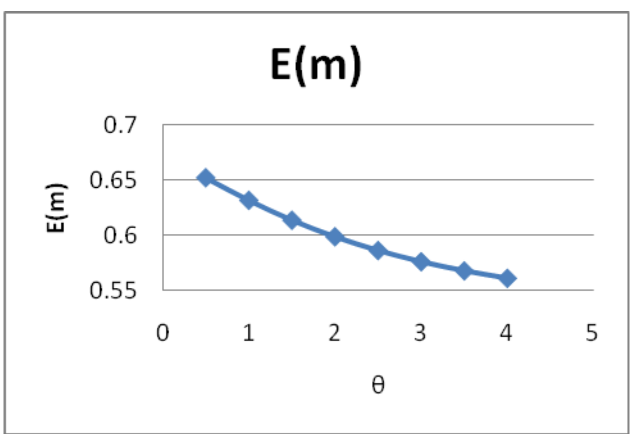

Figure 37. Plot of $E(m)$ vs. θ

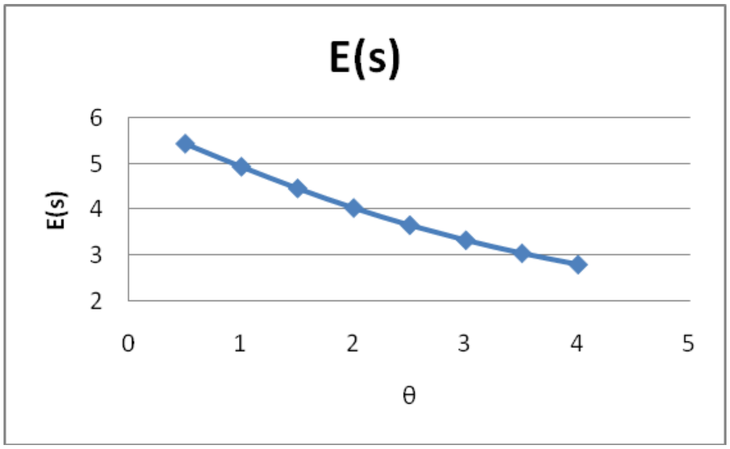

Figure 38. Plot of $E(s)$ vs. θ

Case 8: Varying k and M = [1, 10

Figure 39 shows that $E(m)$ increases from 0.66 when $M = 5$ and $k = 5$ to 0.6886 a day when $M = 10$ and $k = 10$. The relatively high IED clearance rate (both by convoys and clearance units) compared to the placing rate of IEDs decreases the rate that the trucks are being removed for the RS. Figure 40 shows that $E(s)$ is also increasing with both M and k, $E(s)$ increases 4.929 from when $M = 5$ and $k = 5$ to 7.4509 a day when $M = 10$ and $k = 10$.

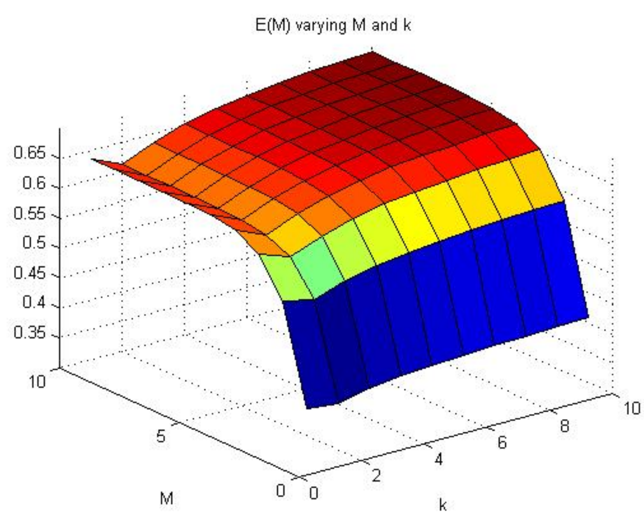

Figure 39. Plot of $E(m)$ vs. M and k

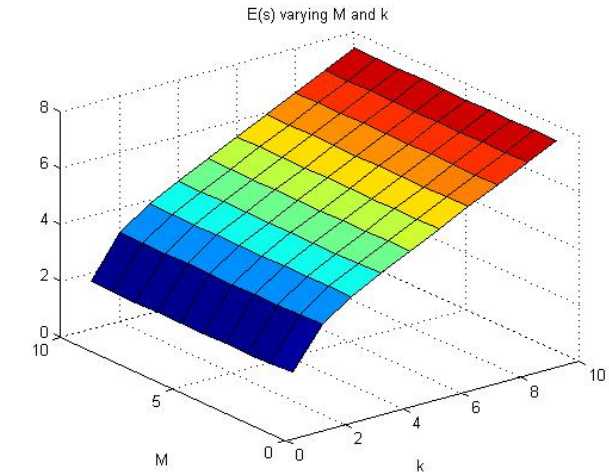

Figure 40. Plot of *E(s)* vs. *M* and *k*

Case 9: Varying μ_1 and μ_2 = [1,15]

Figure 41 shows that *E(m)* is decreasing when μ_2 is increasing while μ_1 is at almost a constant plane for all values of μ_2. The value of *E(m)* decreases from 0.2756 when μ_1 = 5 and μ_2 = 5 to 0.1406 when both parameters double to 10.

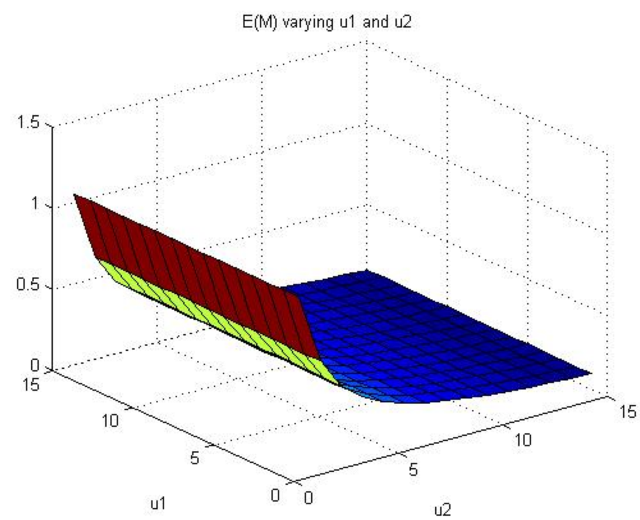

Figure 41. Plot of *E(m)* vs. μ_1 and μ_2

Figure 42 shows that it is the opposite for the values of E*(s)*, that is *E(s)* increasing when μ_1 is increasing while μ_2 is at almost at a constant plane for all values of μ_2. The value of *E(s)* increases from 5.7764 when μ_1 = 5 and μ_2 = 5 to 5.8871 when both parameters double to 10. Therefore, it is important for Blue to invest in technology to improve the IED' detecting and neutralizing rate to reduce the loss of convoys and successful delivery of supplies.

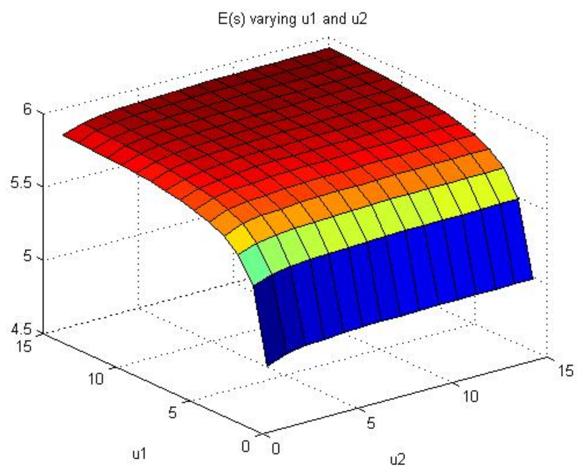

Figure 42. Plot of *E(s)* vs. μ_1 and μ_2

Case 10: Varying α = [0,1] and μ_1 = [1,15]

Figure 43 shows that *E(m)* decreasing with *α* in a plane. μ_1 does not have a significant impact in the values of *E(m)*. The values of *E(m)* decrease from 0.6742 when α = 0 and μ_1 = 5 to 0.5827 n α = 1 and μ_1 = 15. Figure 43 shows that *E(s)* is increasing when μ_1 is increasing, while α does not have much significant impact in the value of *E(s)*. The value of *E(s)* increases from 5 when α = 0 and μ_1 = 1 to 5.8055 when α = 1 and μ_1 = 15. This increase is mainly due to Red being unable to place enough IED in the RS even though the probability of hitting the convoy is high.

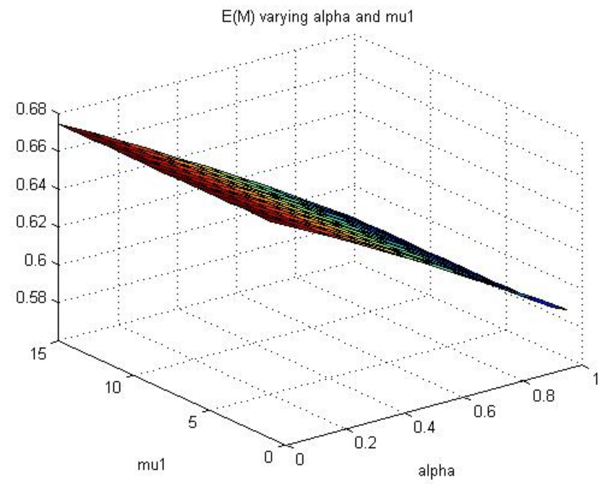

Figure 43. Plot of *E(m)* vs. α and μ_1

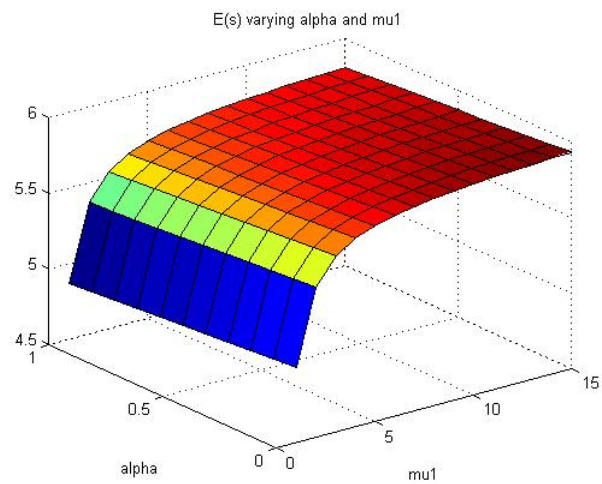

Figure 44. Plot of *E(s)* vs. α and μ_1

Case 11: Varying p = [0,1] and μ_2 = [1,15]

Figure 45 shows that *E(m)* is decreasing when p and μ_2 are increasing. The values of *E(m)* is at a constant value of 2.511 when p = 0 for all values of μ_{2}, and the lowest value of *E(m)* is 0.0662, which is at p = 1 and μ_2 = 15 (that is when both the

probability of detecting and neutralizing the IEDs and rate of the clearance unit is clearing the IEDs are at their highest). Given this information, Figure 46 shows that *E(s)* is also at its peak when the value of *E(m)* is at its lowest.

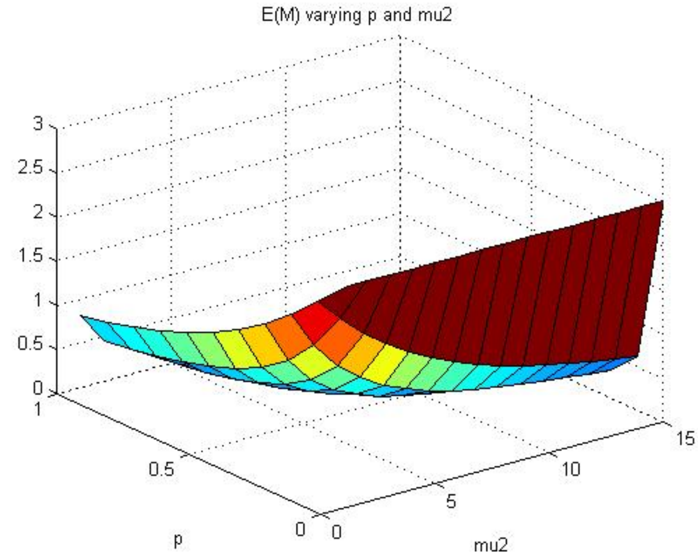

Figure 45. Plot of E(m) vs. p and μ_2

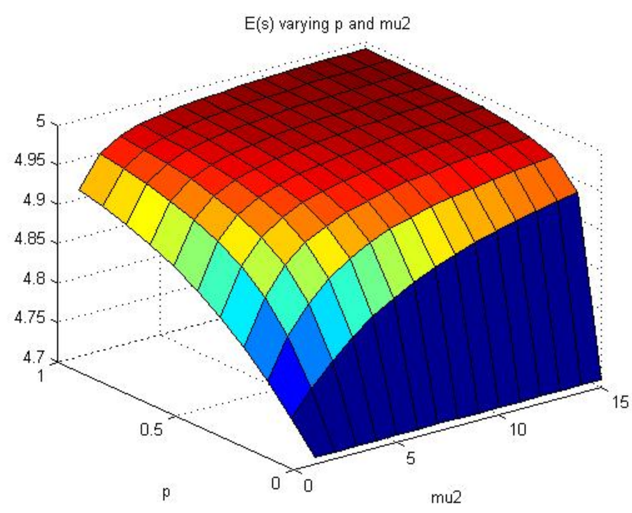

Figure 46. Plot of E(s) vs. p and μ_1

Case 12: Varying σ and θ = [1, 15]

Figure 47 shows that *E(m)* decreases from 0.5416 when σ = 1 and θ = 1 to 0.6348 when σ = 10 and θ = 10. The value of *E(m)* seems to have leveled off when both σ and θ are at their higher values. This is evidenced in Figure 48, as *E(s)* also level off when both σ and θ are at their higher values.

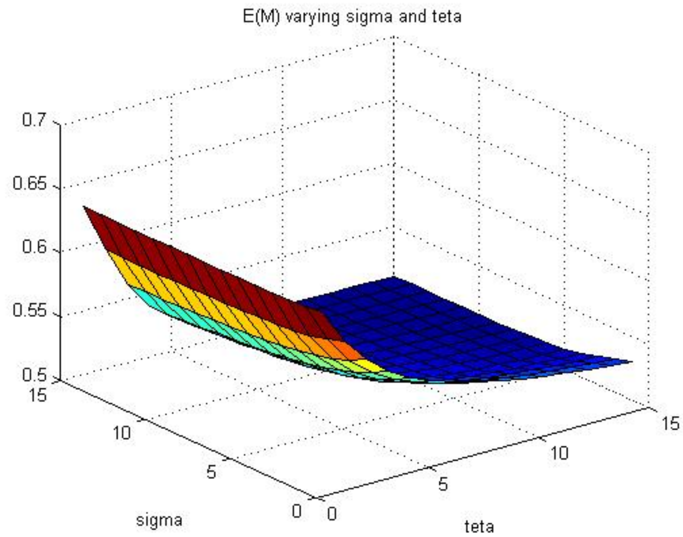

Figure 47. Plot of *E(m)* vs. σ and θ

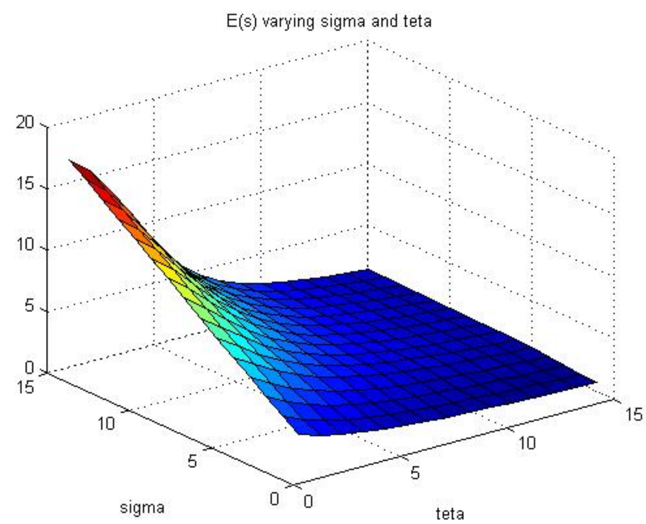

Figure 48. Plot of *E(s)* vs. σ and θ

V. CONCLUSION AND RECOMMENDATIONS

In this chapter, we highlight the key conclusions and recommendations that are derived from the Markov models presented in this thesis.

A. PERSISTENT SHOOTING TACTICS USING SALVOS

From the model developed with the choice of base case in Chapter II, we can conclude that both the number of salvos fired, x, and the number of rounds fired in each salvo has a significant impact on the expected number of killed targets, especially when both of these parameters are kept small because the expected number of killed targets will reach steady state when a huge number of munitions are fired. In practice, we do not need to eliminate all the VTs; killing up to a threshold may disable the enemy's effectiveness. When the number of available munitions is limited, it is concluded that a higher number of smaller salvos is preferred to firing a large quantity of round in one or two salvos as it gives a higher expected number of killed targets.

The model also shows that both the probability of BDA and the probability of kill play a significant role in expected number of killed targets. As such, the following courses of action are recommended:

- Deliver higher number of munitions with small number of salvos as this will allow a higher expected rate of attrition with munitions preservation.
- Improve both the probability of kill in each salvo and the probability of BDA as these two factors can greatly increase the expected rate of attrition.

B. TRANSPORTATION TACTICS IN THE PRESENCE OF IEDS

Based on the model and choice of base case, which reflects a reasonable scenario, it is concluded that beyond a relatively small number ($M = 5$) of IEDs, the IED capacity of the RS does not affect the steady state attrition rate. The IED deployment rate by Red, λ, has a relatively strong influence the attrition rate. Also, the rate of convoys entering the road segment, μ_1, and the probability of hitting the convoy, α, have a significant

effect on increasing the steady state attrition rate. The two factors that help reduce the steady state attrition rate are the rate at which friendly forces detect and neutralize the IEDs, μ_2 , and the probability of detecting and neutralizing the IEDs, *p*.

As such, the following courses of action are recommended:

- Limit the rate of convoys travelling through the road segment by designating rendezvous points to muster the convoys before they are dispatched. However, this is highly dependent on the schedule of the supplies needed.
- Improve the rate of dispatching clearance units and the probability of detection and neutralizing. Improving these two factors can greatly reduce the attrition rate of convoys.

C. SUPPLY MODEL IN THE PRESENCE OF IEDS

Building on the basic model that was discussed in the above section with inventory control, similar to the results in the previous section, the IED deployment rate by Red, λ, and the IED capacity of the RS, *M*, increases the expected number of IEDs. The rate of convoys entering the road segment, μ_1, does not has a significant effect on increasing the expected number of IEDs in the RS. The factors that help reduce the expected number of IEDs in the RS are the rate at which friendly forces detect and neutralize the IEDs, μ_2 , order point in the destination point, σ, and the destination point inventory demand rate, θ.

However, a counterintuitive trend is observed when the expected number of IEDs decreases while the number of trucks passing through the RS increases. However, this trend may be explained by the fact that longer convoys result in a smaller number of dispatches and thus fewer convoys-IED encounters.

The expected number of supplies in the destination decreases when the IED deployment rate by Red, λ, the IED capacity of the RS, M, and the destination point inventory demand rate, θ, increases. The rate of convoys entering the road segment, μ_1, the number of truck passing the RS, k, the rate at which friendly force detect and neutralize the IEDs, μ_2, and order point in the destination point, σ, help increase the expected number of supplies in the destination point.

As such, the following courses of action are recommended to maintain the inventory level while minimizing risk.

- Similar to the previous section, limit the rate of convoys travelling through the road segment by designating rendezvous points to muster the convoys before they are dispatched. However, this is highly dependent on the schedule of the supplies needed.
- Increase the number of trucks in each convoy to deliver the supply thus reducing the frequency that the convoys need to be sent out.
- Fix the order point in the destination level to a higher level by keeping a larger inventory, thus minimizing the number of trip the convoys need to take to supply the destination. However, this is not usually possible, especially with daily necessities.

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